Reduced-Basis Method for Maxwell's Equations: Understanding Electromagnetics via Circuit Theory

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1 Reduced-Basis Method in Electromagnetics

2 Circuit Theory in Electromagnetics







1 Reduced-Basis Method in Electromagnetics

2 Circuit Theory in Electromagnetics



- Full-wave design of microwave circuits.
- We had a ROM for fast frequency sweep that no one used for design.
- It worked but you never knew the frequency band of convergence.
- You never knew how many moments you have to solve for in the circuit you were dealing with.
- In the end, only a well-trained expert could make it worthy for FFS.

Goals

- Reliable reduced-order model.
- Efficient model order reduction process.
- No expertise help be required.

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Time-Harmonic Maxwell's Equations. Weak formulation

Continuous problem

Find
$$\overline{H} \in \mathcal{X}$$
 such that

$$\int_{\Omega} \left(\frac{1}{\varepsilon_r} \nabla \times \overline{H} \cdot \nabla \times \overline{W} - k^2 \mu_r \overline{H} \cdot \overline{W} \right) dv = -\frac{jk}{\eta_0} \int_{\partial\Omega} \overline{\Psi} \cdot \overline{W} ds \quad \forall \overline{W} \in \mathcal{X}.$$

 \mathcal{X} is $H(curl;\Omega)$.

Finite Element formulation

Find
$$\overline{H} \in \mathcal{X}^{\mathcal{N}}$$
 such that
$$\int_{\Omega} \left(\frac{1}{\varepsilon_r} \nabla \times \overline{H} \cdot \nabla \times \overline{W} - \mathbf{k}^2 \mu_r \overline{H} \cdot \overline{W} \right) dv = -\frac{j\mathbf{k}}{\eta_0} \int_{\partial\Omega} \overline{\Psi} \cdot \overline{W} ds \quad \forall \overline{W} \in \mathcal{X}^{\mathcal{N}}$$

 $\mathcal{X}^\mathcal{N}$ is Nédélec finite elements.

Reduced basis space



CEM

We tend to approximate the field solution itself \implies Large approximation space \mathcal{X}

BUT...

The field does NOT arbitrarily vary. It evolves on reduced manifold \mathcal{M}_k

WHY NOT?

Try a hand at approximating this low-dimensional manifold instead? Reduced-basis space.

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Reduced basis space. Proper Orthogonal Decomposition

Proper orthogonal decomposition

- Fine sampling of the field solutions in the parameter space.
- O Throw away all linearly dependency in the field solutions via SVD.



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Problem statement

Original problem. Dimension $M, M \gg 1$

Find $\mathbf{x}(k)$ such that: $\mathbf{A}(k)\mathbf{x}(k) = \mathbf{b}(k) \quad \forall k \in [k_1, k_2]$

Reduced-order model. Dimension $n, n \ll M$

Find
$$\widetilde{\mathbf{x}}(k)$$
 such that:
 $\mathbf{V}^T \mathbf{A}(k) \mathbf{V} \widetilde{\mathbf{x}}(k) = \mathbf{V}^T \mathbf{b}(k)$ $\mathbf{V} = cols\{\mathbf{v}_1, \dots, \mathbf{v}_n\}$
 $\widetilde{\mathbf{A}}(k) \widetilde{\mathbf{x}}(k) = \widetilde{\mathbf{b}}(k)$ $\forall k \in [k_1, k_2]$

$\mathbf{A}(k) = \mathbf{S} - k^2 \mathbf{T}$ $\mathbf{b}(k) = f(k)\mathbf{b}$	$\begin{split} \widetilde{\mathbf{A}}(k) &= \widetilde{\mathbf{S}} - k^2 \widetilde{\mathbf{T}} \\ \widetilde{\mathbf{b}}(k) &= f(k) \widetilde{\mathbf{b}} \\ \widetilde{\mathbf{S}} &= \mathbf{V}^T \mathbf{S} \mathbf{V}, \ \widetilde{\mathbf{T}} = \mathbf{V}^T \mathbf{T} \mathbf{V}, \ \widetilde{\mathbf{b}} = \mathbf{V}^T \mathbf{b} \end{split}$

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Affine parameter dependence	Derived parameter dependence
$\mathbf{A}(k) = \mathbf{S} - \frac{k^2 \mathbf{T}}{\mathbf{b}(k)} = \frac{f(k)\mathbf{b}}{\mathbf{b}}$	$ \begin{split} \widetilde{\mathbf{A}}(k) &= \widetilde{\mathbf{S}} - \frac{k^2 \widetilde{\mathbf{T}}}{\widetilde{\mathbf{b}}(k)} \\ \widetilde{\mathbf{b}}(k) &= \frac{f(k) \widetilde{\mathbf{b}}}{\widetilde{\mathbf{S}}} \\ \widetilde{\mathbf{S}} &= \mathbf{V}^T \mathbf{S} \mathbf{V}, \ \widetilde{\mathbf{T}} &= \mathbf{V}^T \mathbf{T} \mathbf{V}, \ \widetilde{\mathbf{b}} = \mathbf{V}^T \mathbf{b} \end{split} $

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A posteriori error estimation

$$\widetilde{\mathbf{A}}(k)\widetilde{\mathbf{x}}(k) = \widetilde{\mathbf{b}}(k) \qquad \qquad \mathbf{x}(k) \simeq \mathbf{V}\widetilde{\mathbf{x}}(k) = \sum_{i} \widetilde{x_i}(k)\mathbf{v}_i$$

Residual

$$\mathbf{R}(\widetilde{\mathbf{x}}(k);k) = \mathbf{b}(k) - \mathbf{A}(k)\mathbf{V}\widetilde{\mathbf{x}}(k) = \mathbf{b}(k) - \sum_{i}\widetilde{x}_{i}(k)\mathbf{A}(k)\mathbf{v}_{i}$$

Affine parameter dependence

$$\mathbf{R}(\widetilde{\mathbf{x}}(k);k) = f(k)\mathbf{b} - \sum_{i}\widetilde{x}_{i}(k)\mathbf{S}\mathbf{v}_{i} + k^{2}\sum_{i}\widetilde{x}_{i}(k)\mathbf{T}\mathbf{v}_{i}$$

 $\|\mathbf{R}(\widetilde{\mathbf{x}}(k);k)\|^2 = (\mathbf{R}(\widetilde{\mathbf{x}}(k);k),\mathbf{R}(\widetilde{\mathbf{x}}(k);k))$

 $\left(\mathbf{b},\mathbf{b}\right), \ \left(\mathbf{b},\mathbf{S}\mathbf{v}_{i}\right), \ \left(\mathbf{b},\mathbf{T}\mathbf{v}_{i}\right), \ \left(\mathbf{S}\mathbf{v}_{i},\mathbf{S}\mathbf{v}_{j}\right), \ \left(\mathbf{T}\mathbf{v}_{i},\mathbf{T}\mathbf{v}_{j}\right), \ \left(\mathbf{S}\mathbf{v}_{i},\mathbf{T}\mathbf{v}_{j}\right)$

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Canonical waveguide filter. Frequency band = 14 - 16 GHz



ROM of dimension 3

6 ROM of

Canonical waveguide filter. Frequency band = 14 - 16 GHz



ROM of

dimension 4

6 ROM of

Canonical waveguide filter. Frequency band = 14 - 16 GHz



- 8 ROM of dimension 5
- B ROM of

Canonical waveguide filter. Frequency band = 14 - 16 GHz



- ROM of dimension 6

Canonical waveguide filter. Frequency band = 14 - 16 GHz



- Solution ROM of dimension 7

Canonical waveguide filter. Frequency band = 14 - 16 GHz



O ROM of dimension 8

Canonical waveguide filter. Frequency band = 14 - 16 GHz



- ROM of dimension 3
 ROM of dimension 3
 - dimension 4
- 3 ROM of dimension 5
- ROM of
- SOM of
 - dimension 7
- ROM of dimension 8

O ROM dim = 9_{222}

Canonical waveguide filter

Frequency band = 14 - 16 GHz. Reduced basis dimension = 9

RBM



Canonical waveguide filter

Frequency band = 14 - 16 GHz. Reduced basis dimension = 9

FEM



Frequency band = 1.6 - 1.9 GHz. Reduced basis dimension = 10

RBM



Frequency band = 1.6 - 1.9 GHz. Reduced basis dimension = 10

FEM



Dual-mode waveguide filter

Frequency band = 11.6 - 12 GHz. Reduced basis dimension = 5



[ref] J. R. Montejo-Garai and J. Zapata, "Full-wave design and realization of multicoupled dual-mode circular waveguide filters," IEEE Trans. Microw. Theory Tech., vol. 43, no. 6, pp. 1290-1297, Jun. 1995.

Dual-mode waveguide filter

Frequency band = 10 - 15 GHz. Reduced basis dimension = 14



[ref] J. R. Montejo-Garai and J. Zapata, "Full-wave design and realization of multicoupled dual-mode circular waveguide filters," IEEE Trans. Microw. Theory Tech., vol. 43, no. 6, pp. 1290-1297, Jun. 1995.

5-poles waveguide filter. DDM

[ref] J. Rubio, J. Arroyo, and J. Zapata, "Analysis of passive microwave circuits by using a hybrid 2-D and 3-D finite-element mode-matching method," IEEE Trans. Microw. Theory Tech., vol. 47, no. 9, pp. 1746-1749, Sept. 1999.





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GiØ



4λ diameter sphere



K. Zhao, "A domain decomposition method for solving electrically large electromagnetic problems," PhD Thesis, The Ohio State University, 2007.

Turbine inlet

 $\label{eq:Frequency band} {\sf Frequency \ band} = 1.5 \ {\sf - 2 \ GHz}. \ {\sf Reduced \ basis \ dimension} = 21$

Monostatic RCS

Bistatic RCS



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PEC sphere. 1.5λ - 3λ diameter sphere

Frequency band = 30 - 60 GHz. Reduced basis dimension = 11

Bistatic RCS Mie series

Bistatic RCS RBM



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PEC plate. $\lambda \times \lambda$ - $2\lambda \times 2\lambda$ plate

Frequency band = 30 - 60 GHz. Reduced basis dimension = 15

Monostatic RCS $\theta\theta$

Monostatic RCS $\phi\phi$



Band notched UWB monopole antenna

Frequency band = 2 - 11 GHz. Reduced basis dimension = 23



[ref] J. Martínez-Fernández, V. de la Rubia, J. M. Gil, and J. Zapata, "Frequency notched UWB planar monopole antenna optimization using a finite element method-based approach," IEEE Trans. Antennas Propag., vol. 56, no. 9, pp. 2884-2893, Sep. 2008.

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Image: Image:

Dielectric resonator filter. Circuit synthesis







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Dielectric resonator filter. Zero-pole diagram

Frequency band = 1.6 - 1.9 GHz. Reduced basis dimension = 10+4





Dielectric resonator filter. Zero-pole diagram

Frequency band = 1.6 - 1.9 GHz. Reduced basis dimension = 10+4





- Initial prototype
- **2** Full-wave Simulation. Realible reduced-order model.
- Set the zero-pole diagram and compare to circuit model.
- Aceptable error?
- If not, change geometry and go to step 2.
- If error is OK, we are done!

[ref] P. Kozakowski and M. Mrozowski, "Automated CAD of coupled resonator filters," IEEE Microwave Wireless Compon. Lett., Dec. 2002.

Frequency band = 11.75 - 11.95 GHz.



[ref] J. García, J. Rubio, J. R. Montejo-Garai and J. Zapata, "CAD of cylindrical dielectric resonator filters by a 3-D finite-element segmentation method," Microwave Opt. Technol. Lett., vol. 31, no. 1, pp 71-75, Oct. 2001.

(3)











Frequency band = 11.5 - 12.5 GHz.



[ref] V. de la Rubia, "Reliable reduced-order model for fast frequency sweep in microwave circuits," Electromagnetics, vol. 34, pp 161-170, April 2014.















Reduced-Basis Method in Electromagnetics

2 Circuit Theory in Electromagnetics





Circuit synthesis







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Can we have a better understanding of Electromagnetics?

- Try to get a full-wave coupling matrix from Electromagnetics.
- This coupling matrix is different from the one in filter synthesis. Eletromagnetic phenomena should be taken into account.
- Full-wave coupling matrix can ultimately assist the design process.

How do we do this?

- Try to understand how Electromagnetics behaves.
- We may think each tuning screw has a localized tuning effect on the filter response. Not true.
- Each tuning screw has a global influence. It actually modifies neighbouring couplings!
- We do not have local resonators, namely dielectric resonators. What we actually have are global resonances. But from the circuit theory point of view we'd like to transform them into local resonators.



Global eigenmodes

Electromagnetics in the filter can be represented by N+Peigenmodes. N dominant eigenmodes. N = filter order.

P higher order eigenmodes

Stand for parasitic electromagnetic behaviour.

As a result

A full-wave transversal network arises [ref].

[ref] R. J. Cameron, "Advanced coupling matrix synthesis techniques for microwave filters," IEEE Trans. Microw. Theory Tech., vol. 51, no. 1, pp. 1-10, Jan. 2003.



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Filter topology full-wave coupling matrix

- Rotations are applied to the transversal coupling matrix.
- Additional electromagnetic couplings appear.
- As a result, the coupling route in the actual filter layout arises.
- Global resonances are mapped back to the local resonantors in the filter.
- This is the information we need for CAD.



Dielectric resonator filter. Full-wave coupling matrix

Frequency band = 1.6 - 1.9 GHz. Reduced basis dimension = 10+4



Dielectric resonator filter. Full-wave coupling matrix

Frequency band = 1.6 - 1.9 GHz. Reduced basis dimension = 10+4



- Initial prototype
- **2** Full-wave Simulation. Realible reduced-order model.
- Oupling matrix comparison. Get full-wave circuit model and compare to synthesis design.
- Aceptable error?
- If not, change geometry according to coupling matrix and go step 2.
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Diplexers









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Antenna Arrays



GALILEO system navigation antenna developed at EADS-CASA









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1 Reduced-Basis Method in Electromagnetics

2 Circuit Theory in Electromagnetics



- A fully automated and reliable reduced-order model for fast frequency sweep has been shown.
- ④ A full-wave circuit model for Electromagnetics has been presented.
- Full-wave Zero-Pole diagram and/or full-wave Coupling Matrix CAD approaches have been proposed.
- A better understanding of Electromagnetics is achieved by means of circuit theory.