RESEARCH LABORATORY OF ELECTRONICS
Massachusetts Institute of Technology

Accelerated Integral Equation Methods for the Comprehensive Electromagnetic Analysis of MRI Systems

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MORNET, Bucharest, 20 March 2015

www.rle.mit.edu/cpg

parallel transmission (pTx) project

- pTx project consortium

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CPG @ RLE, MIT
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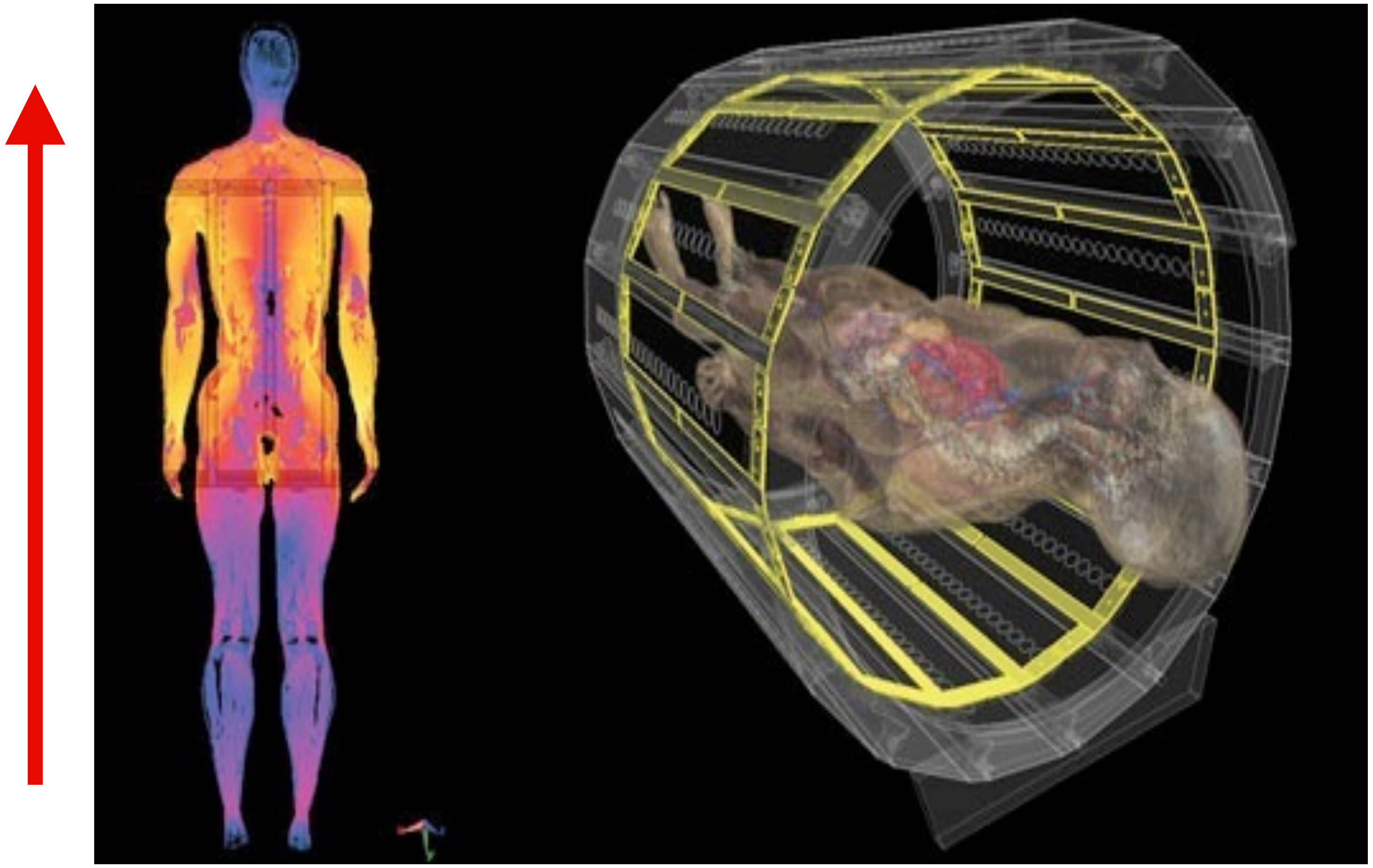
Luca Daniel
Thanos Polimeridis
Zohaib Mahmood
Amit Hochman
Jorge Fernández Villena

But first let's start from the very basics

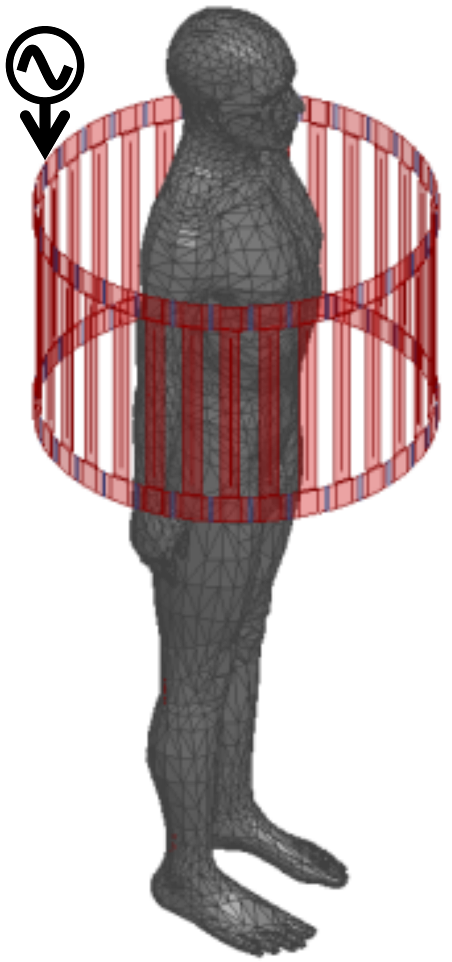
Magnetic Resonance Imaging

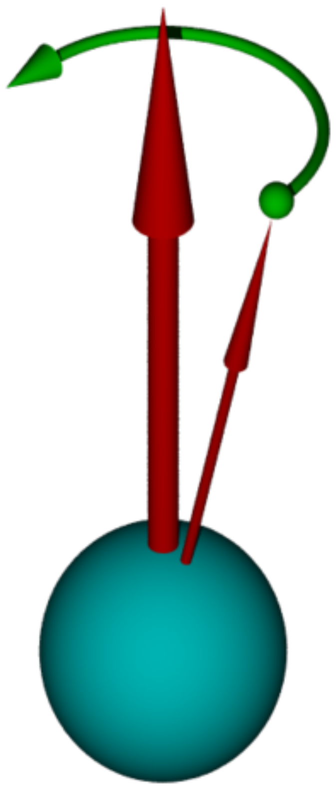
Principles of MRI

- **External large coils generate main uniform B field**
  - align protons in the direction of the scan
  - protons spinning at Larmor frequency
    - multiple of the main field strength



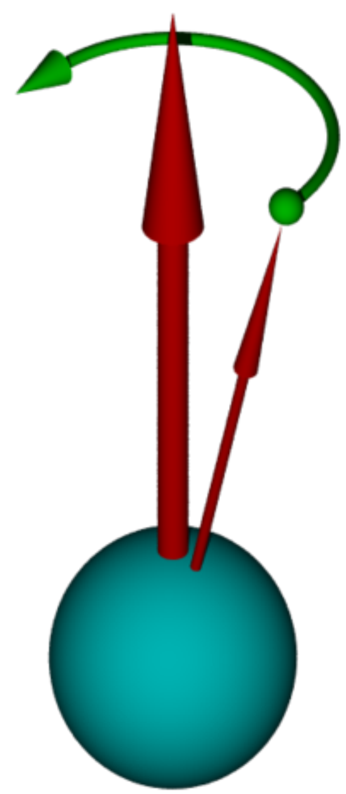
- **RF transmit (Tx) coil**
  - applies a short RF signal at the Larmor frequency
  - perturbes (tilts) the alignment of the spinning

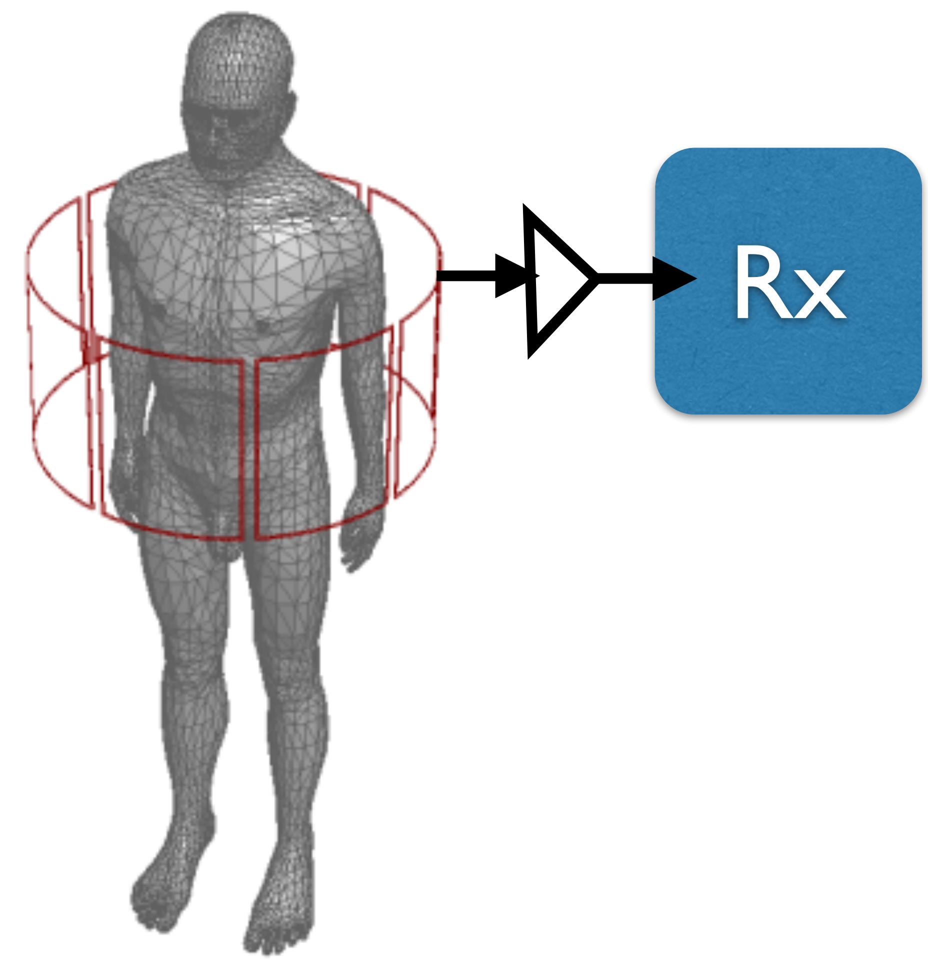




Principles of MRI

- **RF receiving (Rx) coils**
  - protons return to original magnetization alignment
  - generate small flux changes
    - voltage change in receiving (Rx) coils: signal to be processed
  - time to return depends on time constants
    - related to tissue properties
    - contrast in the image





Parallel Transmission

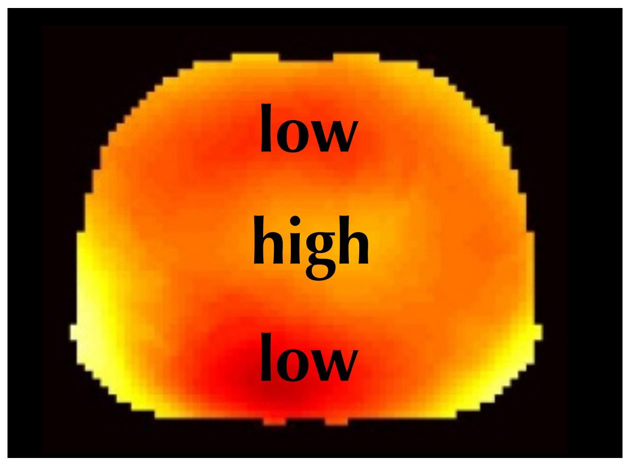
pTx project motivation

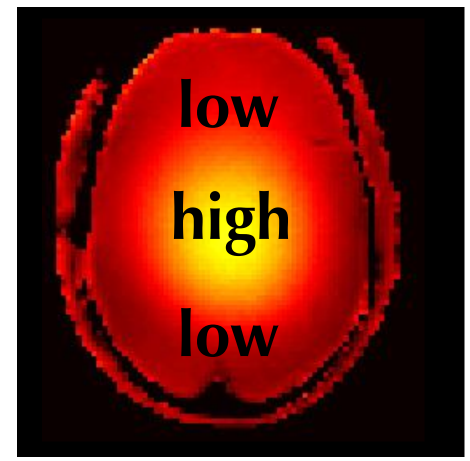
- As we move into high field (higher frequency) scanners

Torso @3T

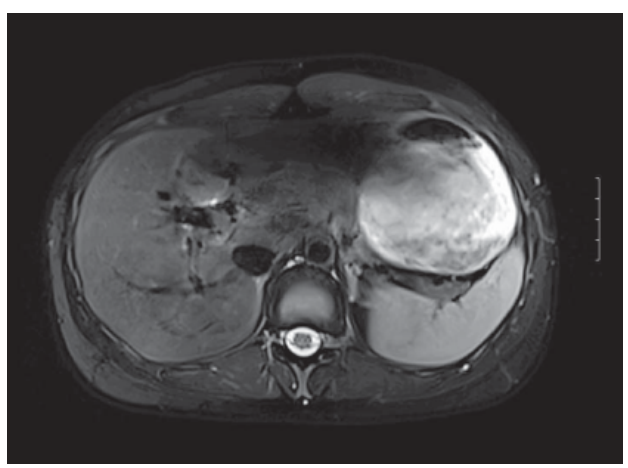
Head @7T

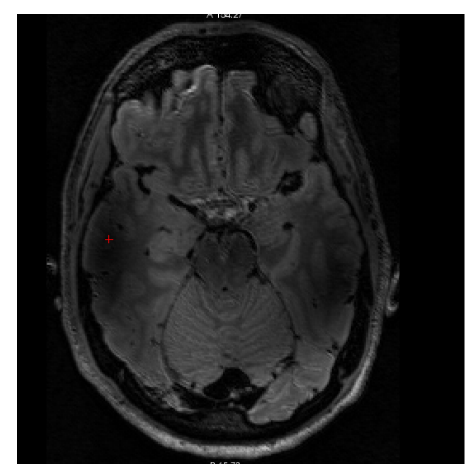
Flip Angle





Image



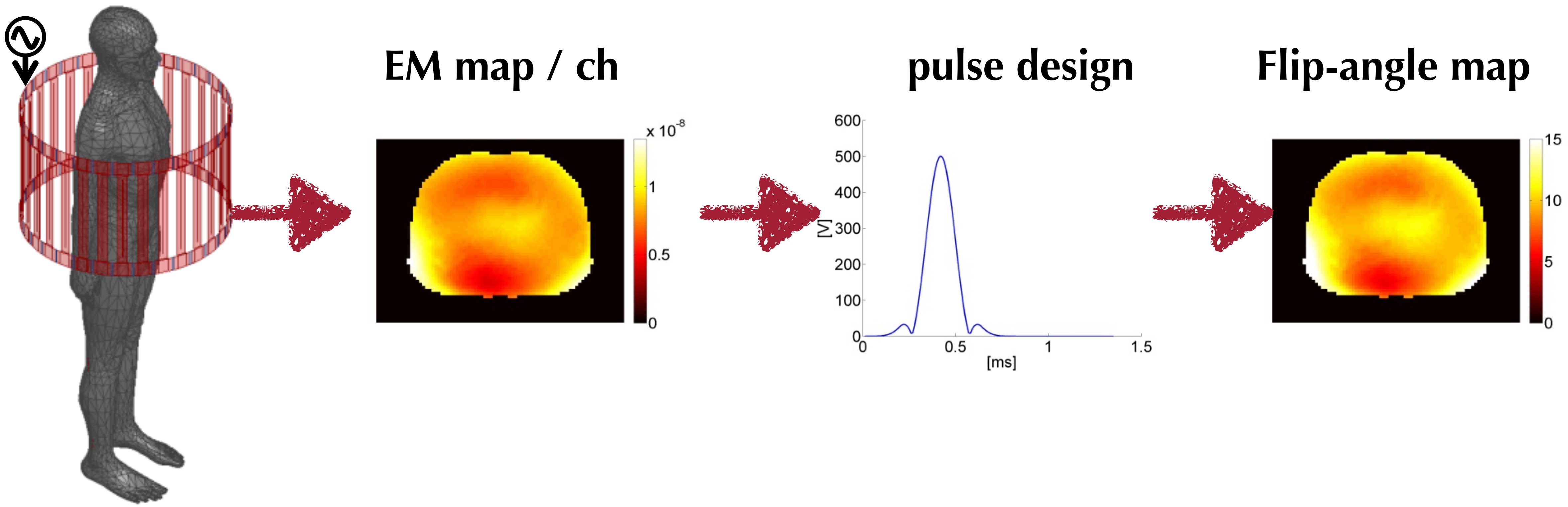


Inhomogeneities affect image quality

Parallel transmission

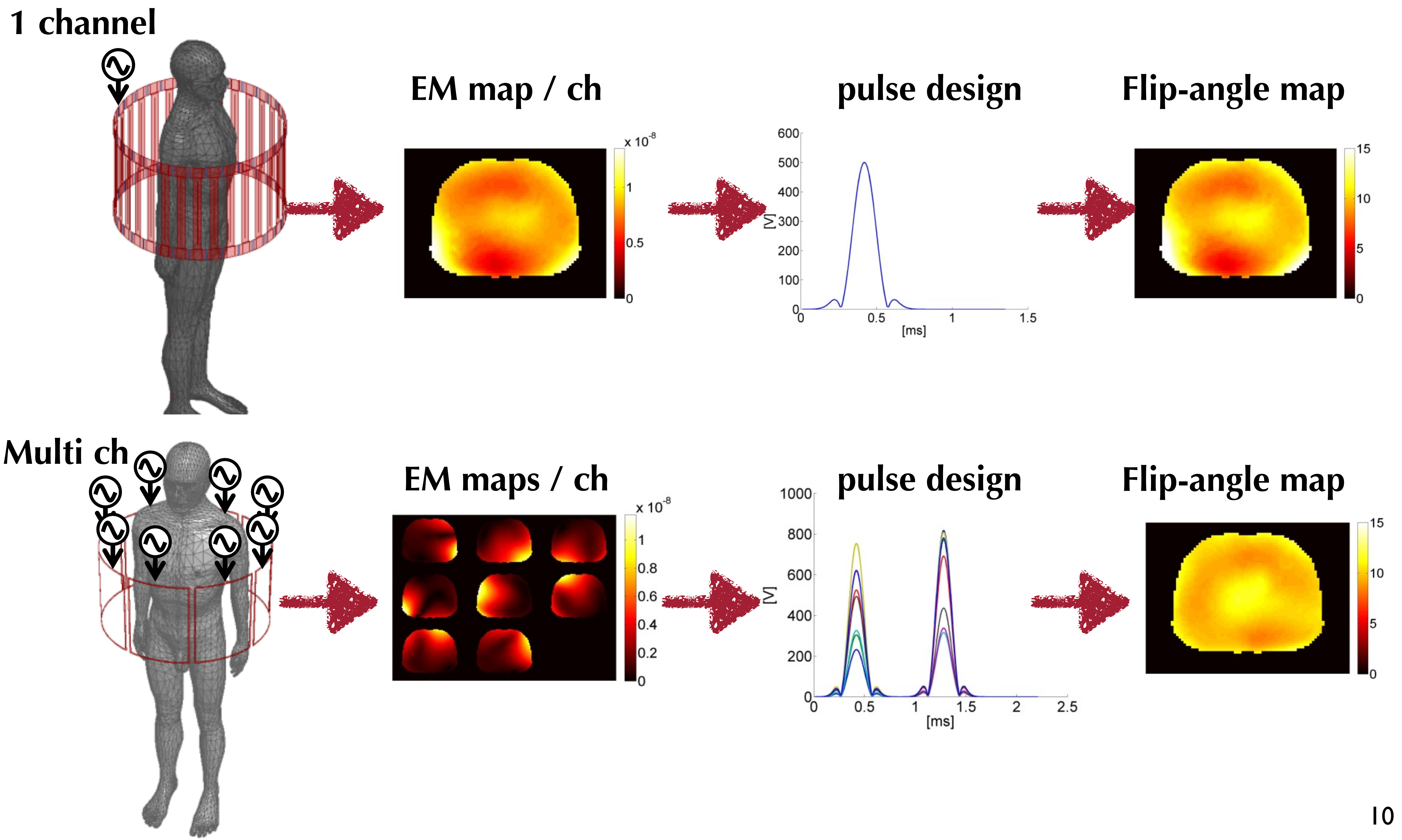
- **Move from single channel**

1 channel



Parallel transmission

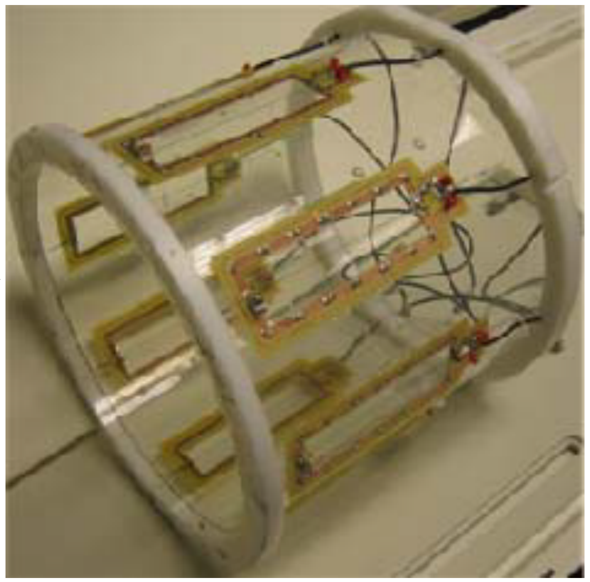
- **Move from single channel to multiple independent channels**



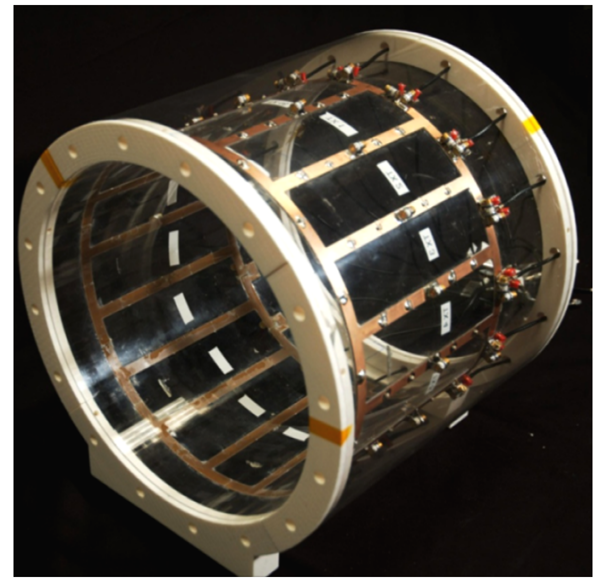
pTx technology

- **New MRI fashion** (images courtesy of Wei Zhao, MGH)

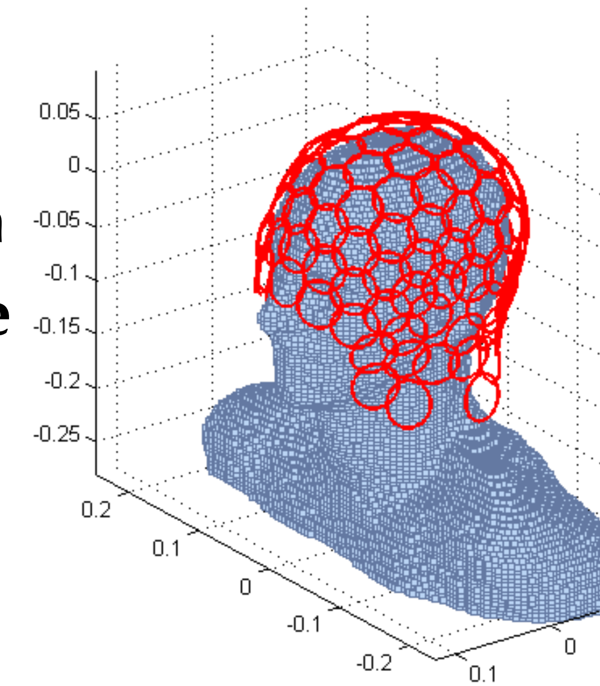
Tx 8ch array



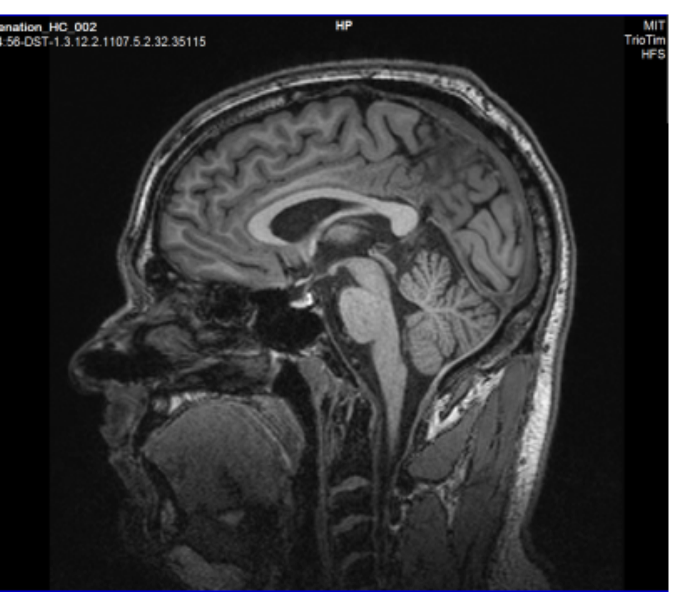
16ch degenerate birdcage

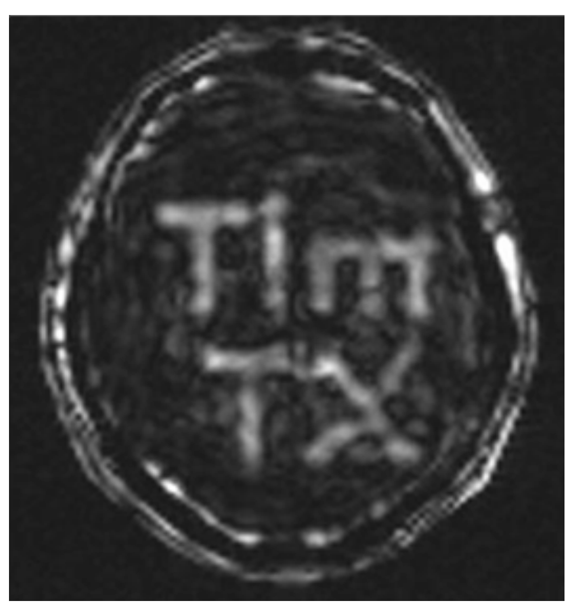


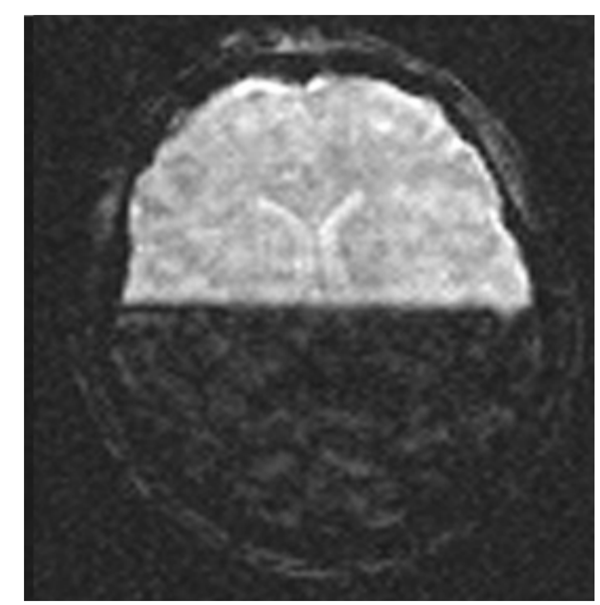
Rx 128ch prototype



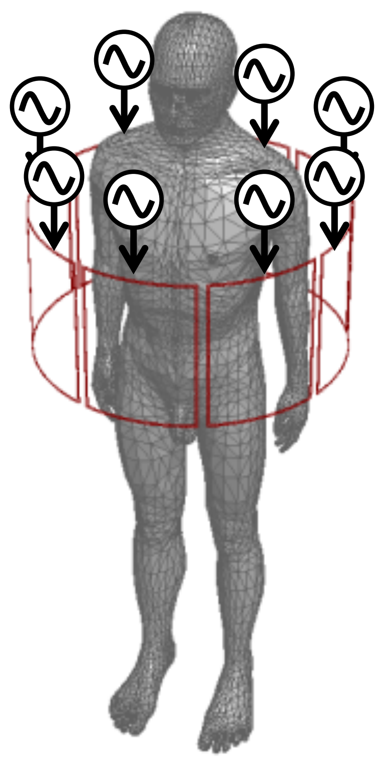
- **and artistic trends**
  - 8ch pTx @7T, MLS (images courtesy of Kawin Setsompop, MGH)

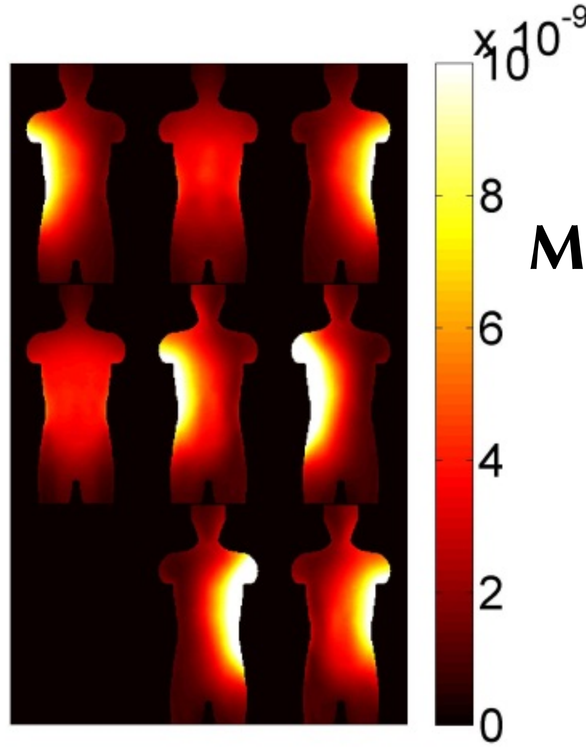




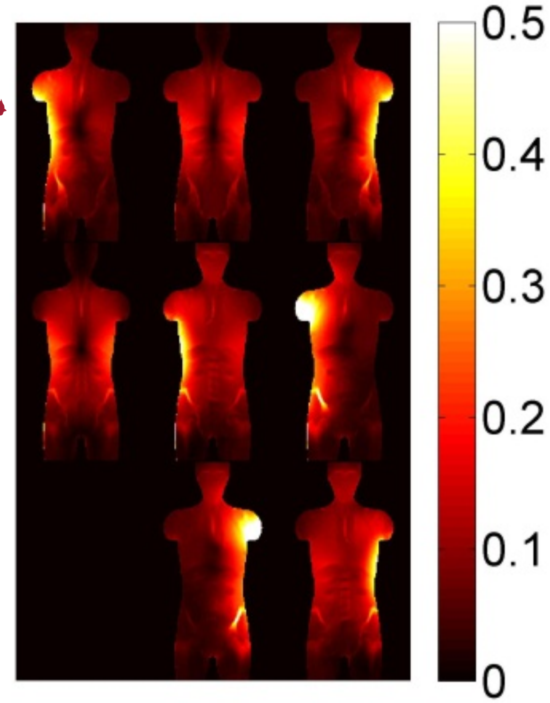


Downside of pTx





Magnetic fields (good) → Improve Image quality



Electric fields (bad) → Increase Energy deposition

SAR must be monitored and limited

- **Specific Absorption Rate (SAR)**
  - time average energy deposed in patient

$$SAR(\mathbf{r}) = \underbrace{\frac{\sigma(\mathbf{r})}{2\rho(\mathbf{r})}}_{\text{tissue properties}} \frac{1}{T} \int_0^T \underbrace{\left\| \mathbf{E}_{tot}(t) \right\|^2}_{\text{detailed E fields}} dt$$

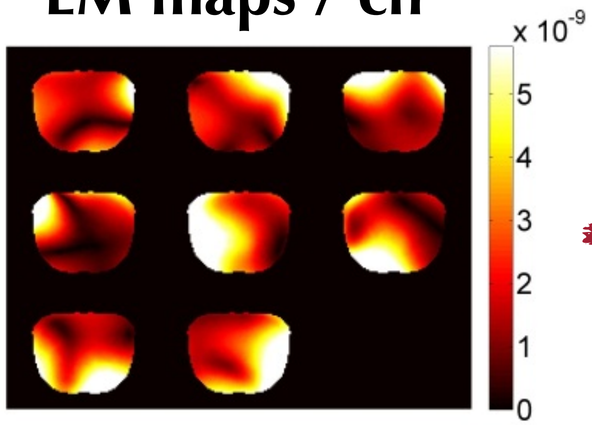
tissue properties

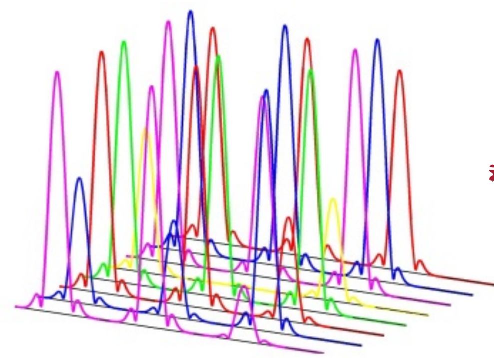
detailed E fields

Reduce SAR

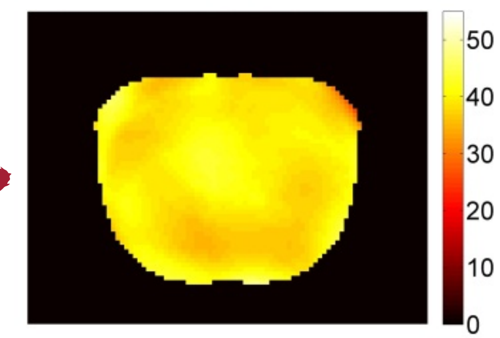
- **How to reduce SAR?**

EM maps / ch | **pulse design** | **Flip-angle** | **SAR**

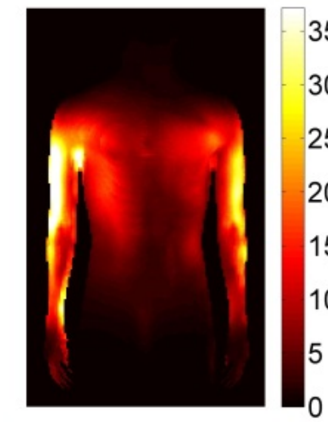




Max 1412 V

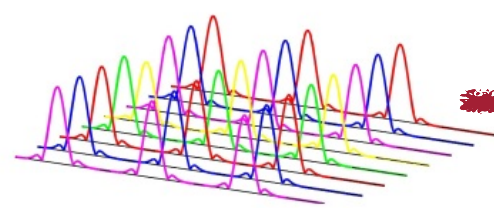


RMSE 13%

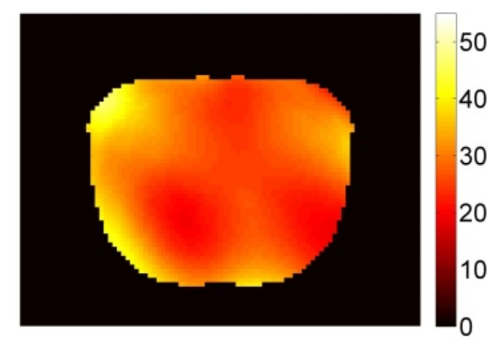


Reduce overall power
SAR < 8W/Kg

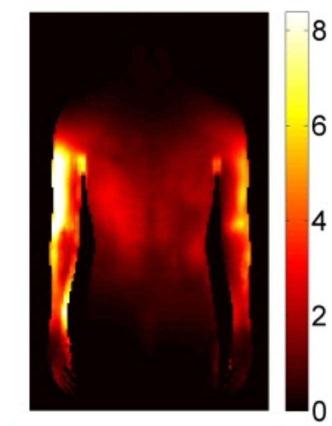
Image quality loss!



Max 447 V



RMSE 39%



Pulse optimization with SAR constrains

- **How to reduce SAR?**

$$SAR(\mathbf{r}) = \frac{\sigma(\mathbf{r})}{2\rho(\mathbf{r})} \frac{1}{T} \int_0^T \left\| \mathbf{E}_{tot}(t) \right\|^2 dt$$

channels

$$\mathbf{E}_{tot}(\mathbf{r}, t) = \sum_{c=1}^{C} \mathbf{b}_c(t) \mathbf{E}_c(\mathbf{r})$$

RF pulse at channel c at time t

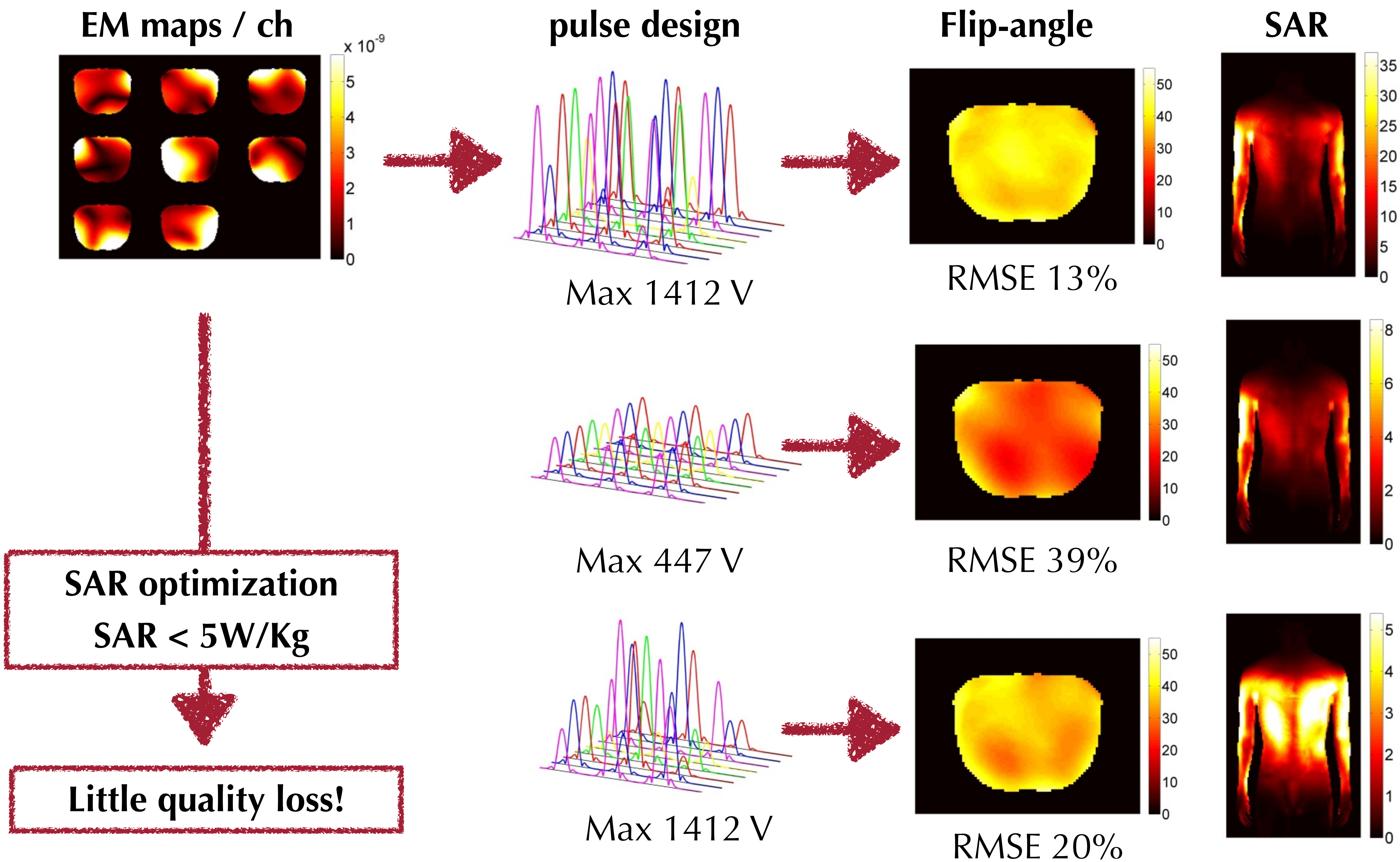
E map of channel c

Play with RF pulses to reduce SAR!

$$SAR(\mathbf{r}) = \frac{1}{N} \sum_{t=1}^{N} \mathbf{b}^H(t) \mathbf{Q}(\mathbf{r}) \mathbf{b}(t)$$

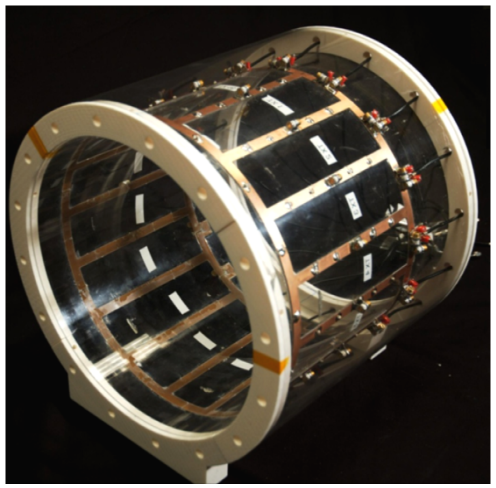
Reduce SAR

- **How to reduce SAR?**

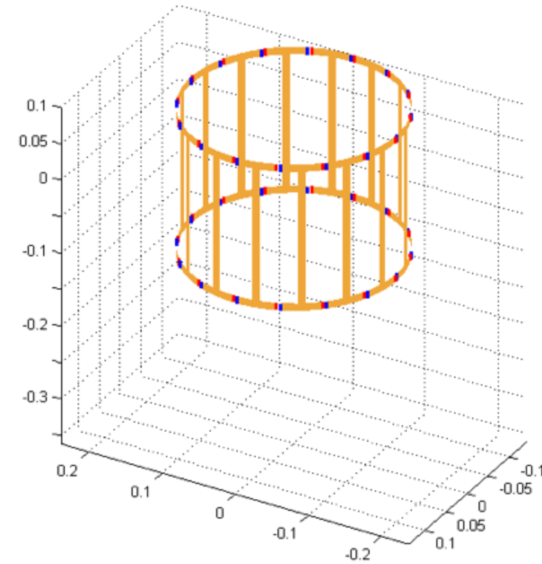


We need the EM field maps per channel

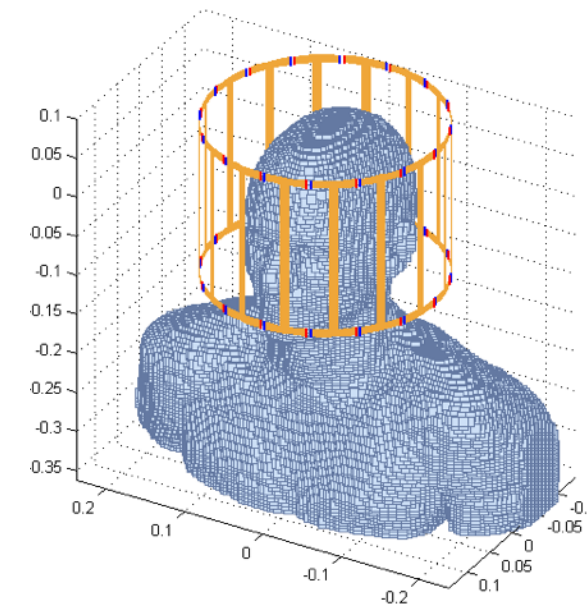
- **Need EM distribution in realistic human body models**



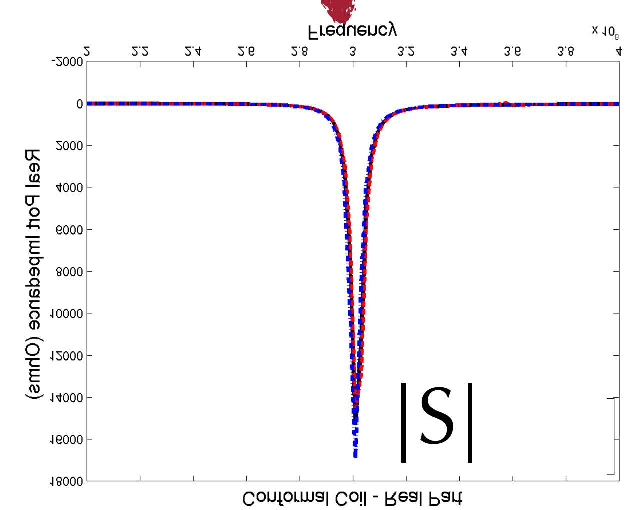
load model
flag ports



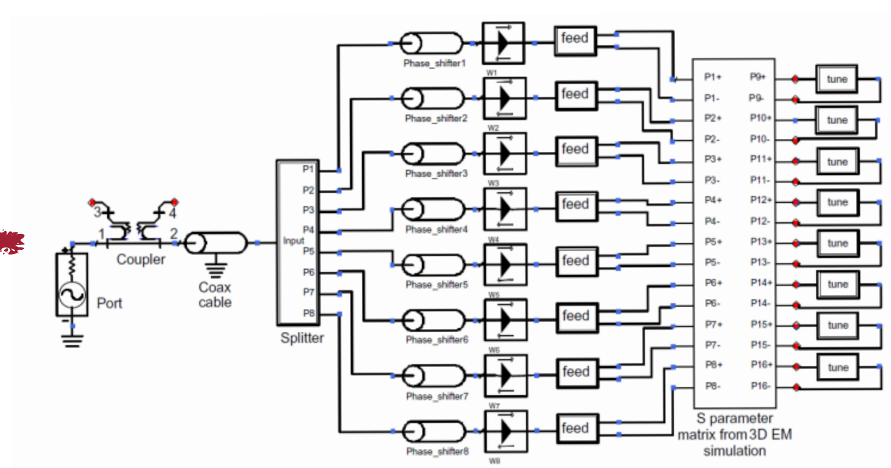
body model



Full-wave EM simulation



circuit co-simulation [1]

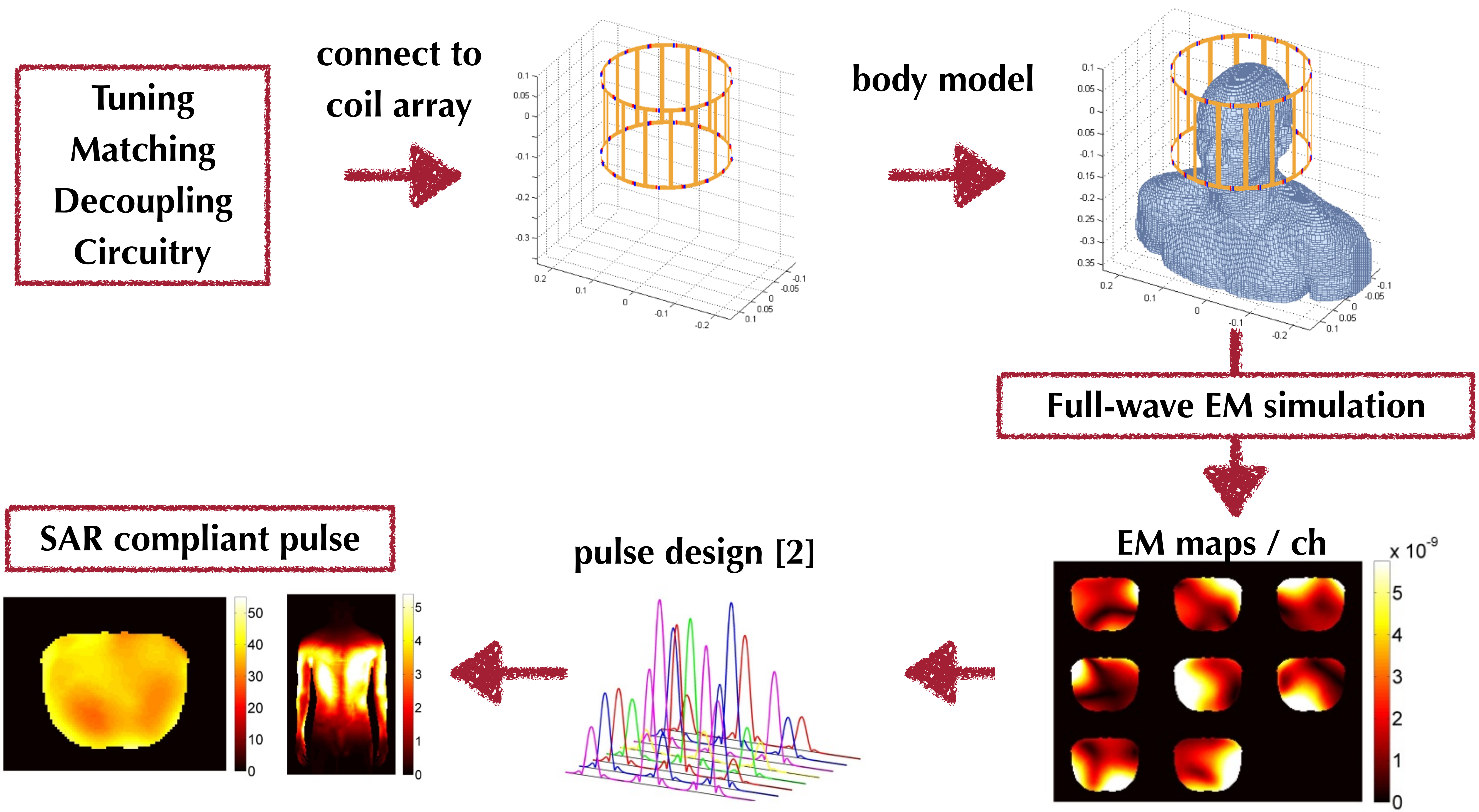


Tuning
Matching
Decoupling
Circuitry

[1] **M. Kozlov et al.**, "Fast MRI coil analysis based on 3-D electromagnetic and RF circuit co-simulation", ***JMR* 2009.**

We need the EM field maps per channel

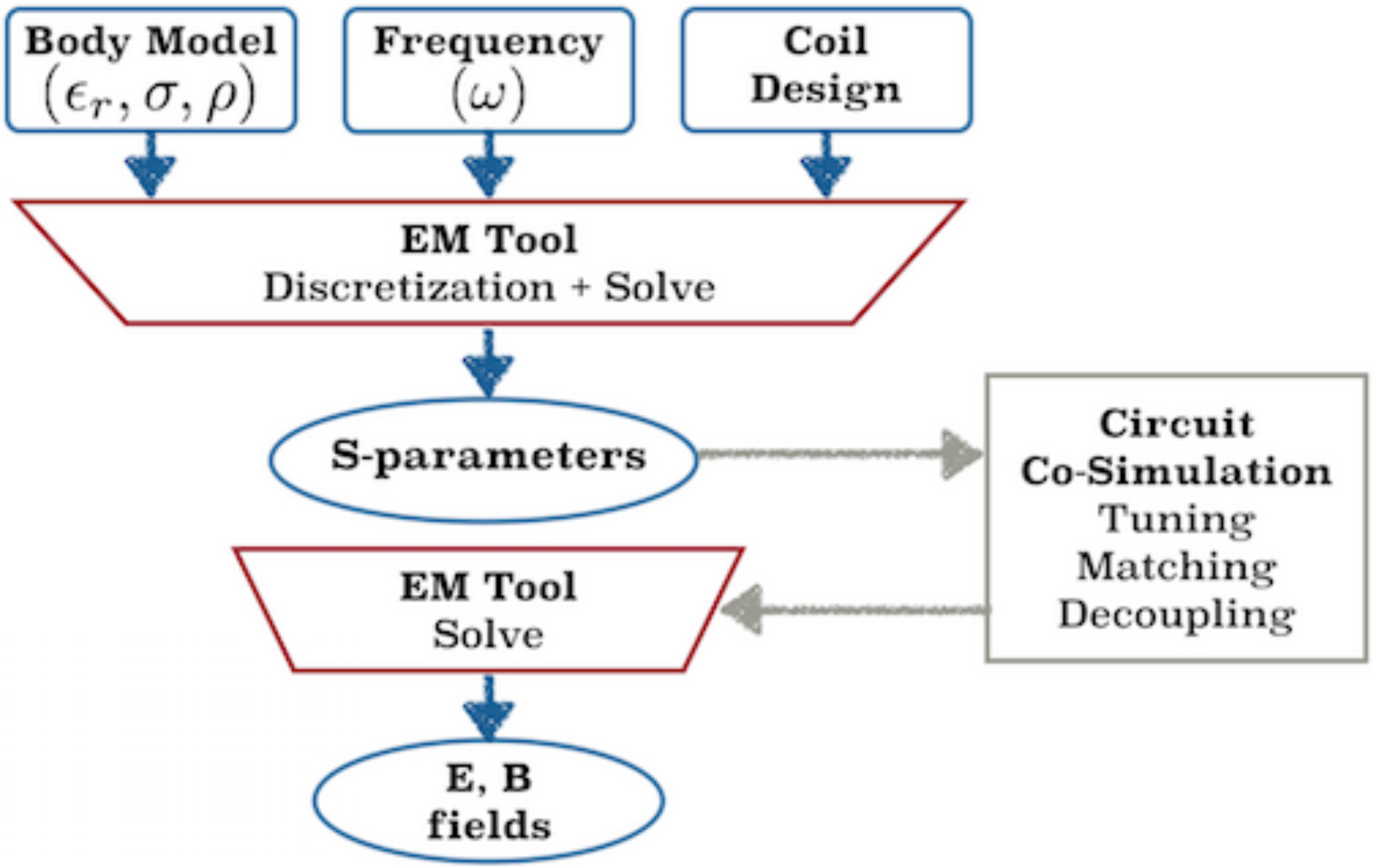
- Need EM distribution in realistic human body models



[2] **B. Guerin et al.**, "Local SAR, global SAR, transmitter power, and excitation accuracy trade-offs in low flip-angle parallel transmit pulse design", **MRM 2013.**

Full-wave EM solver

- **Traditional EM analysis tools**



simply too slow and not flexible!

Traditional solvers

- **EM analysis tools: Surface integral equation methods**
  - Approximate: body model by homogeneous phantom
  - Discretize: only body and conductors surface
    - **model conformal surfaces**
    - **no air discretization**
    - **smaller systems**
    - **fast (10min/simulation)**
    - **OK for coil S-parameters**
    - **approximate EM fields**
    - **useless for SAR calculations**

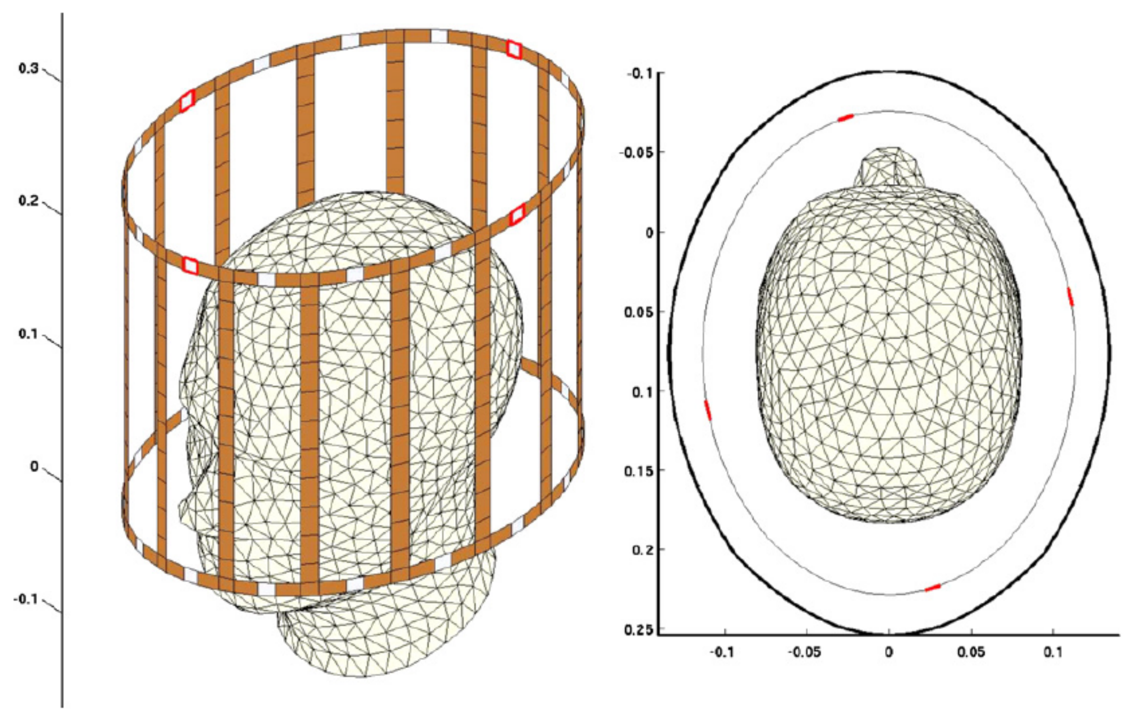


image courtesy of S. Wang, PMB 2008

Traditional solvers

- **EM analysis tools: Finite Difference Time Domain / Finite Element**
  - Volume discretization: inhomogeneous models
  - Discretize: whole domain, plus boundary conditions

    - **refinement for conformal surfaces**
    - **air discretization**
    - **large (sparse) systems**
    - **convergence issues**
    - **Slow (hours/simulation)**
    - **bad coil S-parameters approx. (FDTD)**
    - **good for SAR calculations**

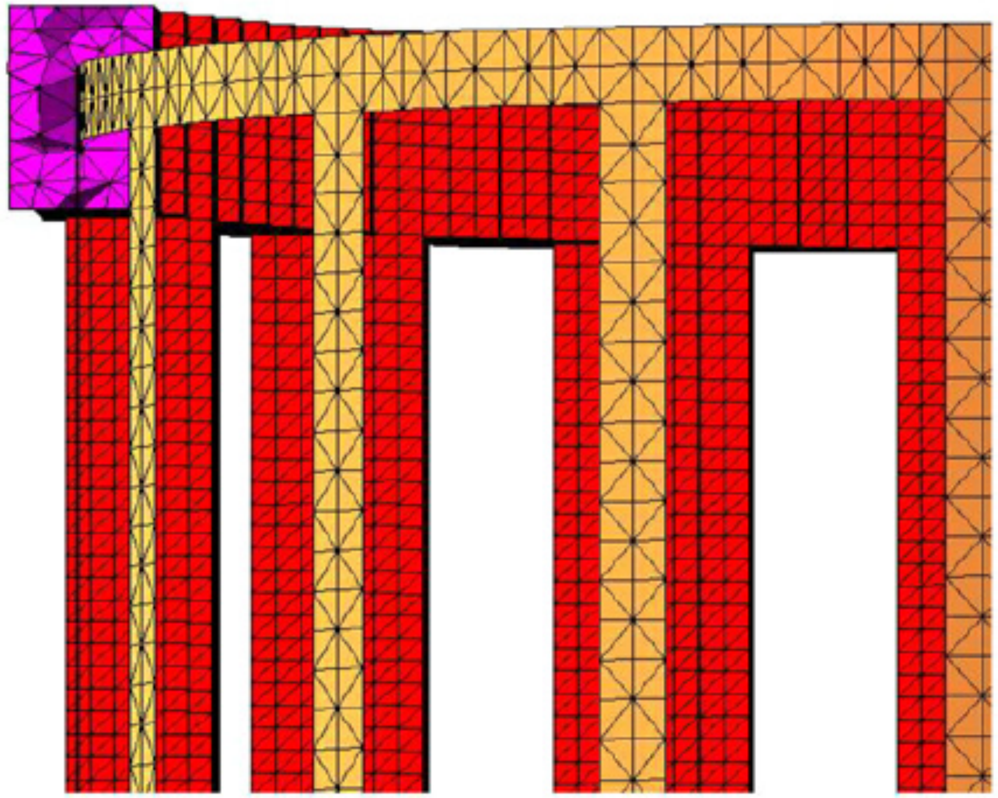


image courtesy of S. Wang, PMB 2008

Brute force acceleration

- **Traditional EM analysis tools: acceleration? sure!**

…for $97 million

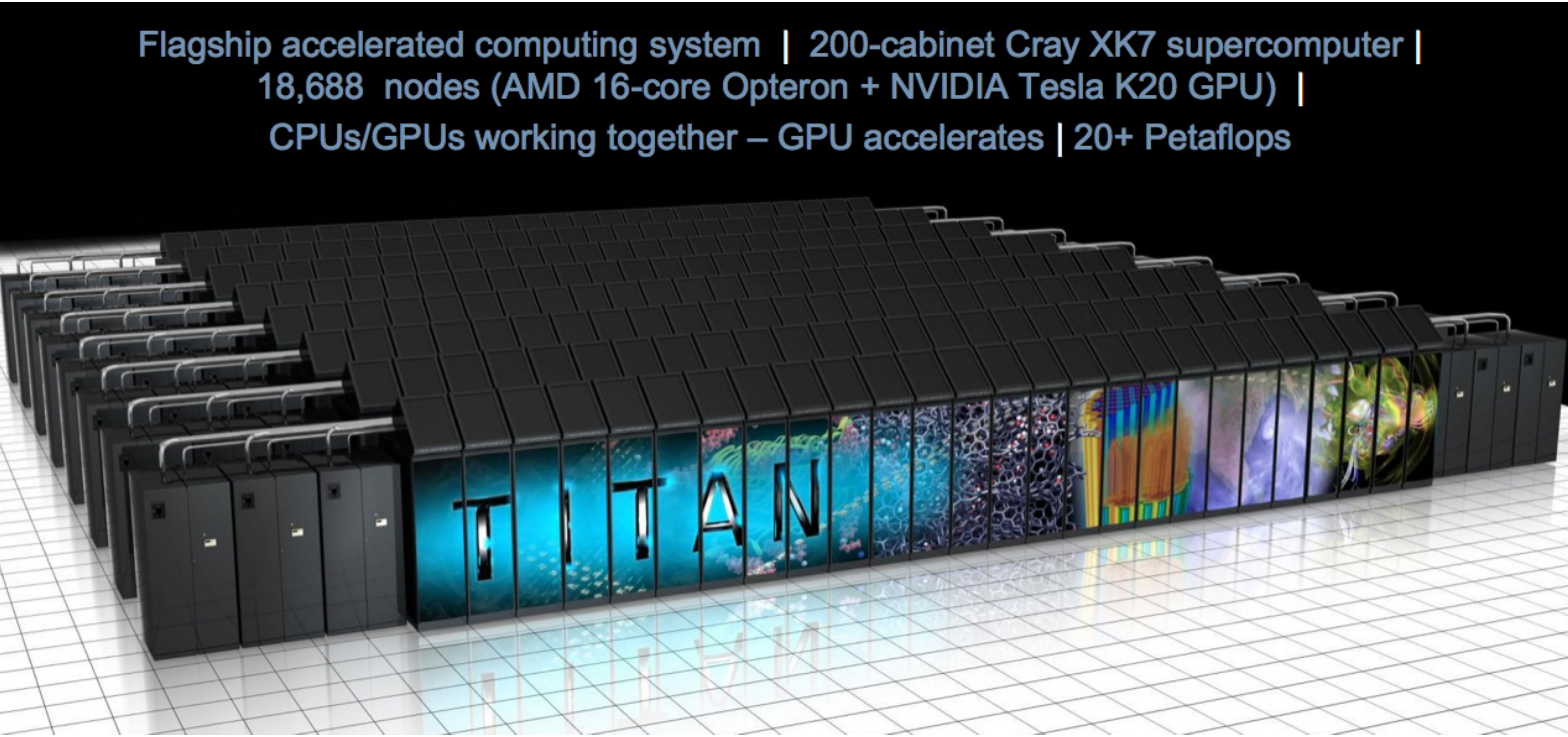
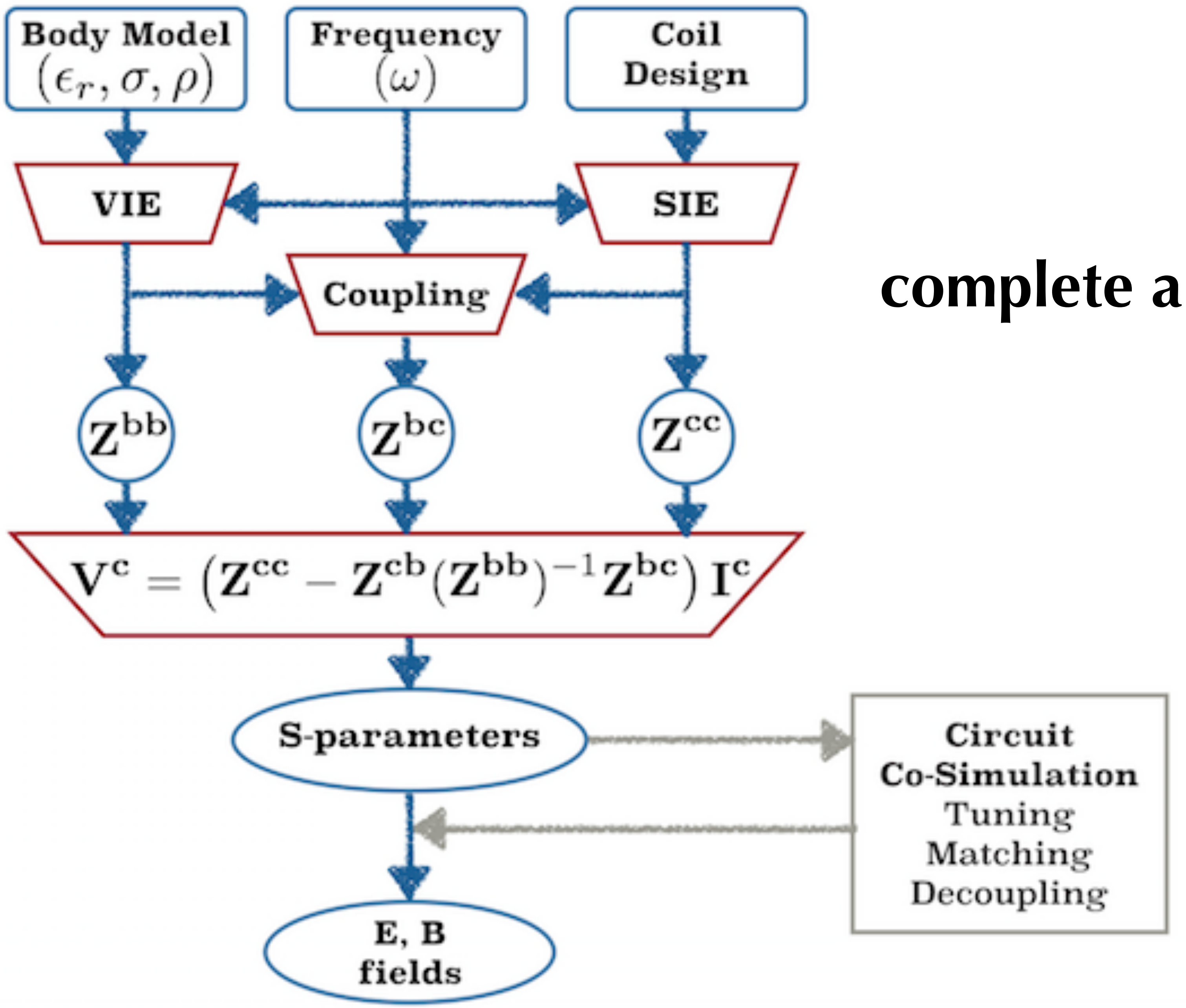


Image: Cray Titan supercomputer at Oak Ridge National Laboratory

CPG role in pTx project

- **Goal: accelerate the EM analysis flow**
  - Integral equation methods & sophisticated numerical methods
  - Leverage problem knowledge



complete analysis in minutes!

Where?

- **1 desktop server**
  - MATLAB 2013 running on Windows R2008
  - two Xeon E2685W (16 cores total) @3.1GHz
  - a K20X GPU Nvidia (6GB mem.)

Goals

Patient Specific MRI

MRI coil design

?

Ultimate SNR/SAR

Robust Optimisation

Goals

Patient Specific MRI

MRI coil design

MARIE

magnetic resonance integral equation

Ultimate SNR/SAR

Robust Optimisation

Public domain

- **Open source MATLAB code**
  - beta-version coming soon

Marie

magnetic resonance integral equation

- **Prototype tools with focus on**
  - Fast solvers for complex inhomogeneous media
  - Combination with surface-based coil models
  - Domain oriented iterative methods
  - Acceleration: pre-computation of fixed parts

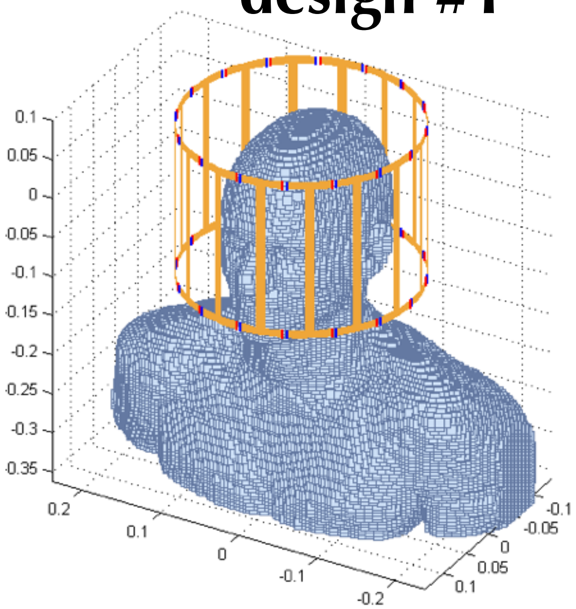
In this presentation

MRI coil design

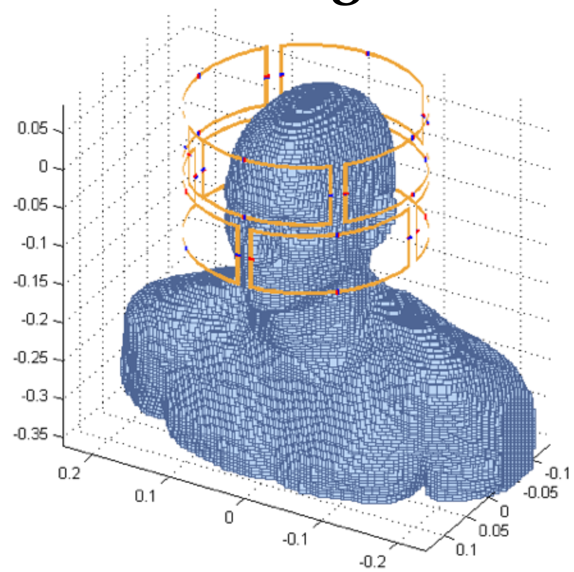
In this presentation…

- **Best out of 500 coil designs?**

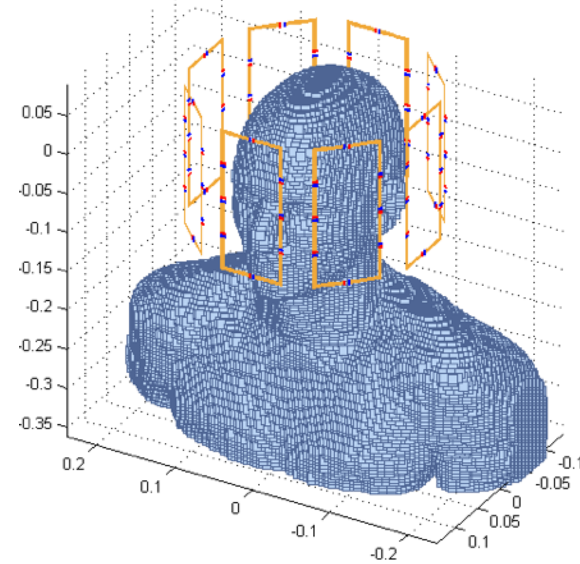
design #1



design #2



design #3



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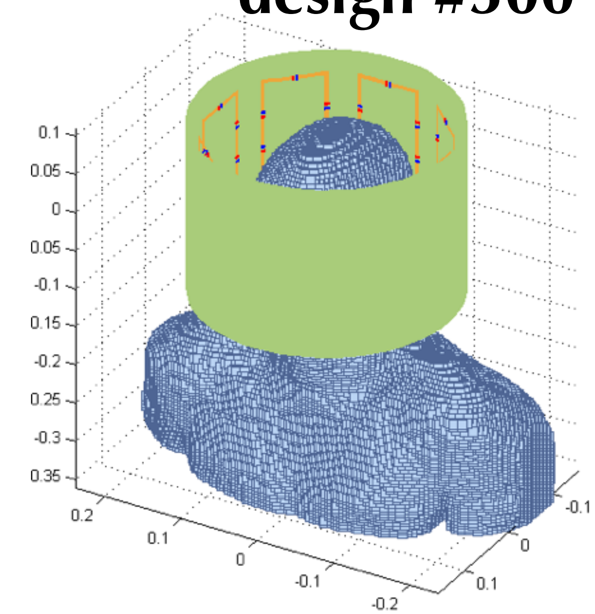
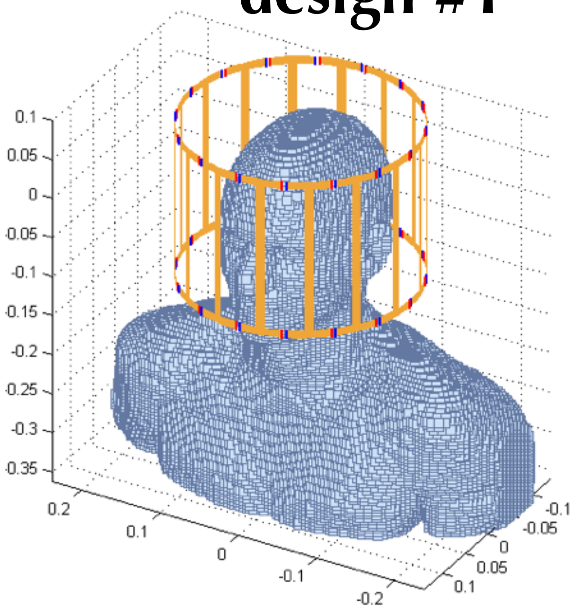
design #500



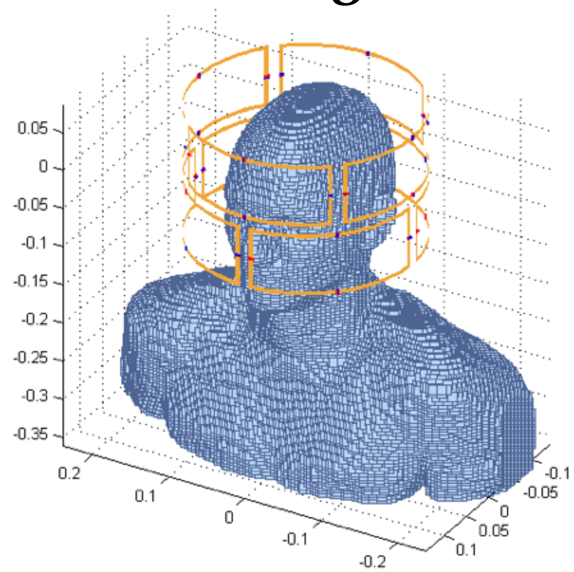
In this presentation…

- **Best out of 500 coil designs?**

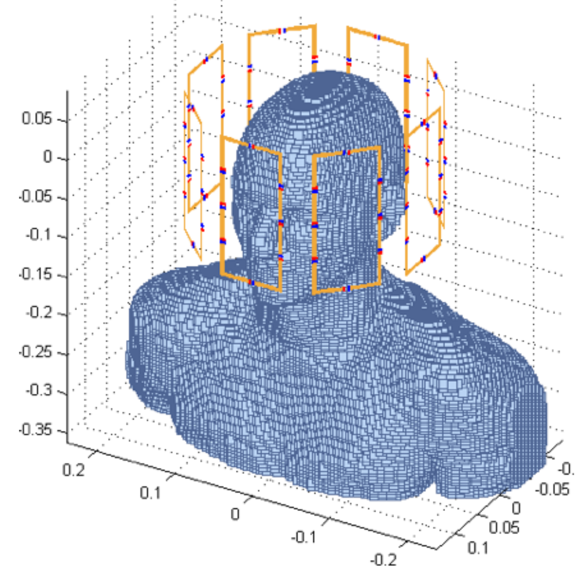
design #1



design #2



design #3



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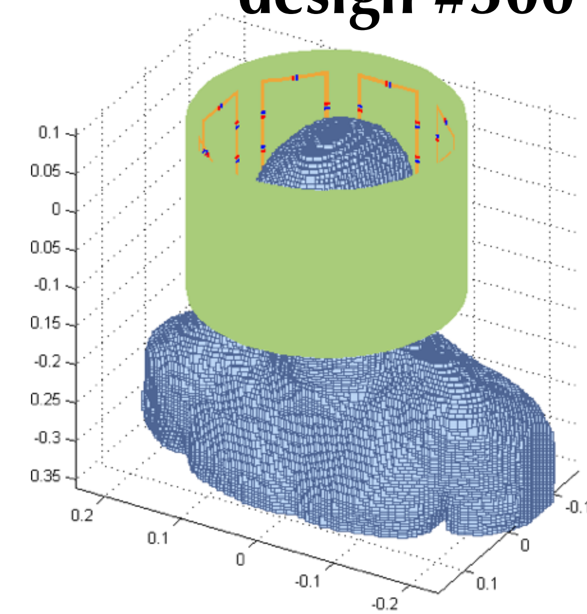
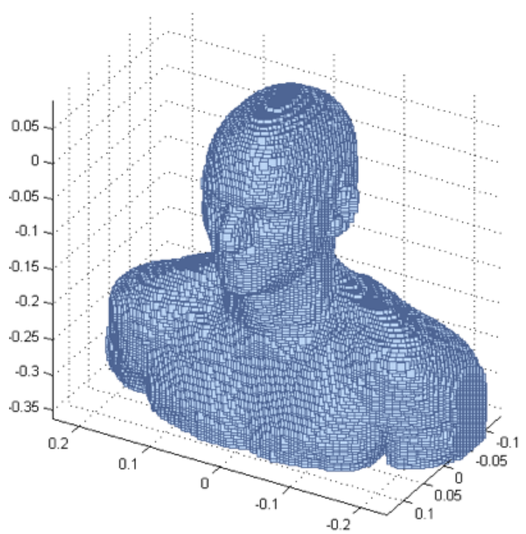
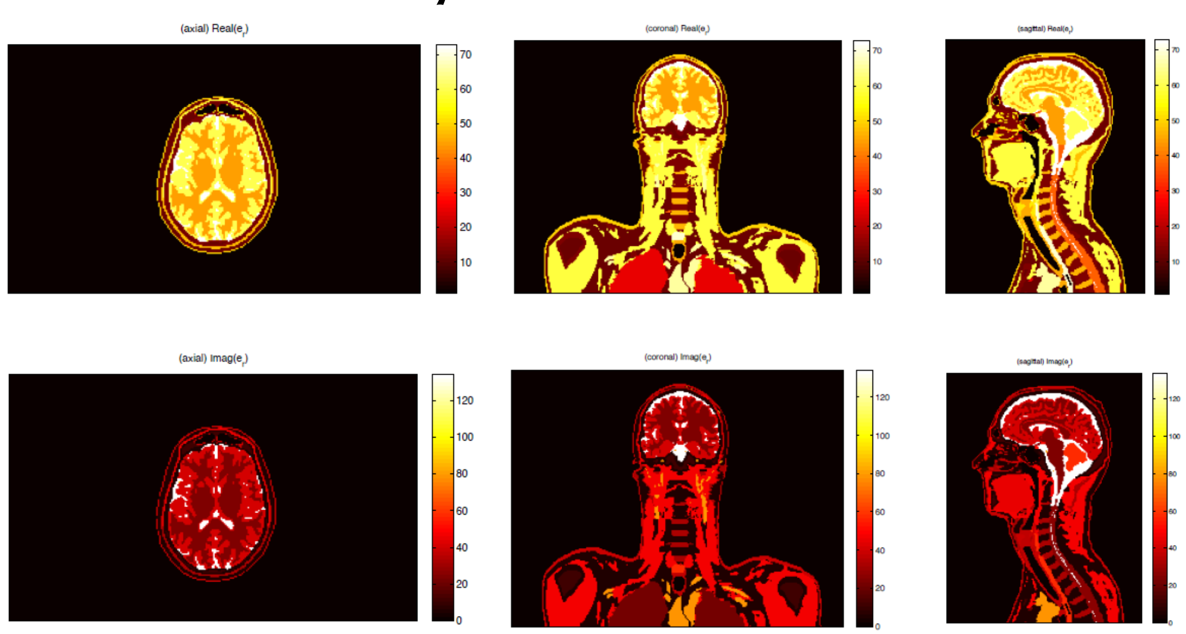
design #500



for inhomogeneous realistic human body models [1]



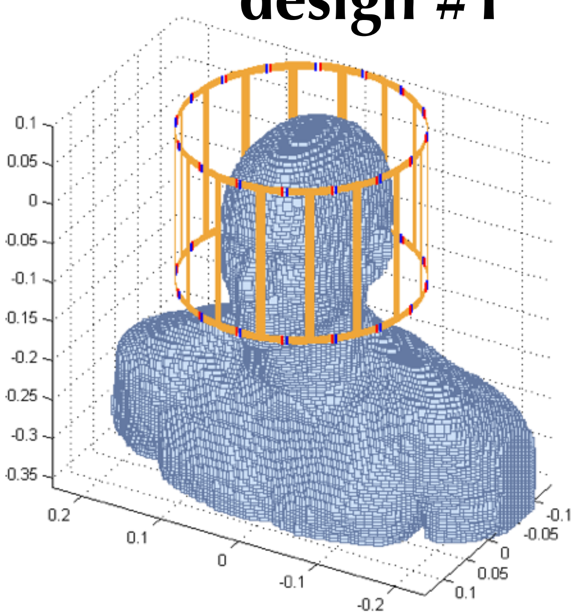


[1] **A. Christ et al.**, "The Virtual Family – development of anatomical CAD models of two adults and two children for dosimetric simulations," ***PMB*** **2010.**

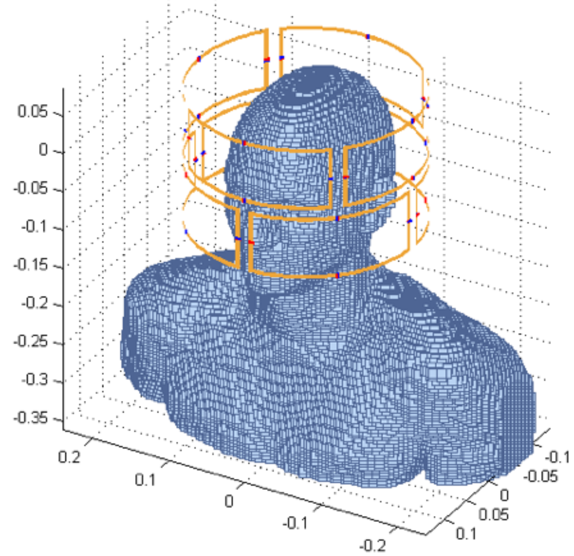
Impact coil design

- **Best out of 500 coil designs?** **Need full EM analysis of each design**

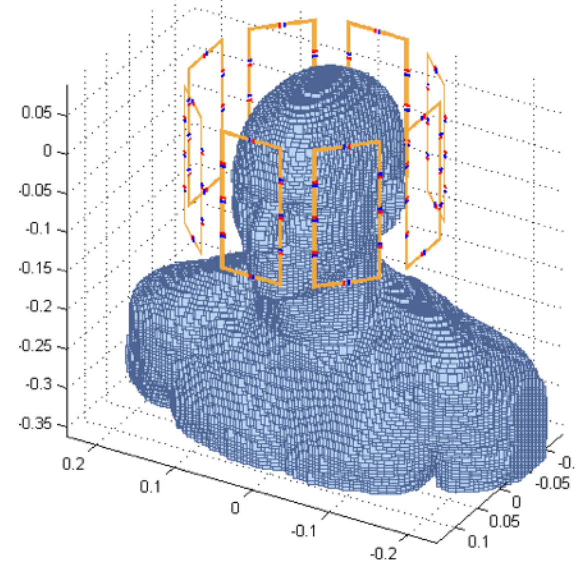
design #1



design #2



design #3



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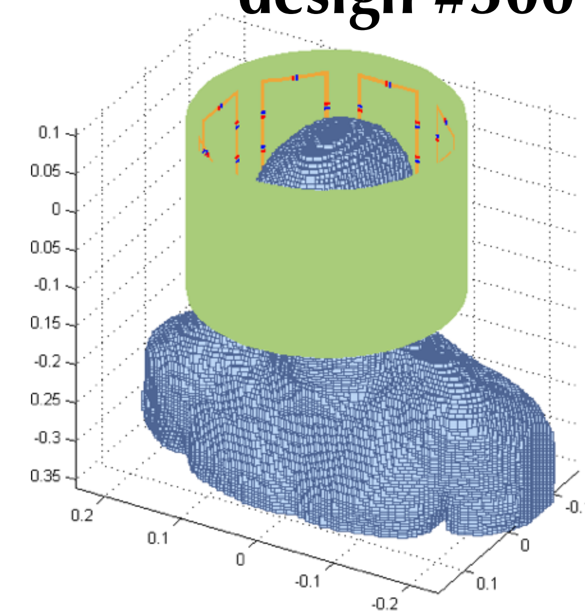
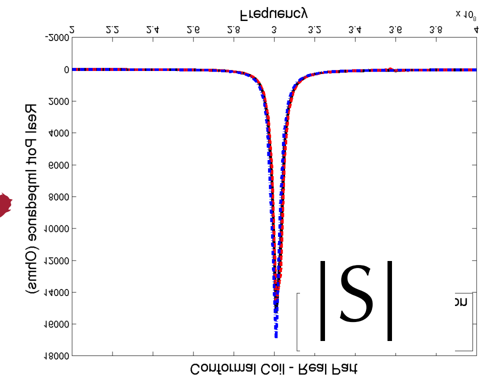
design #500



Full EM analysis



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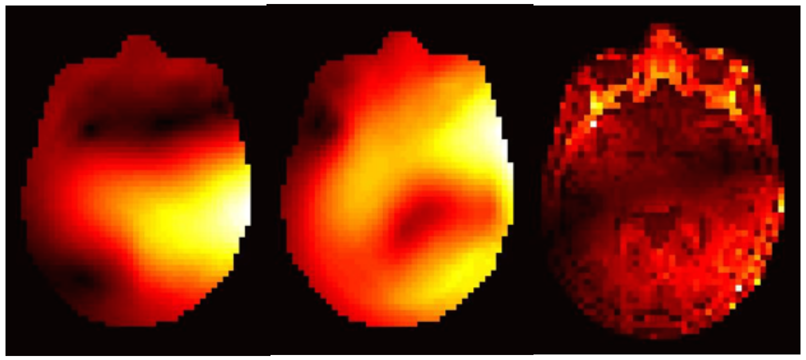
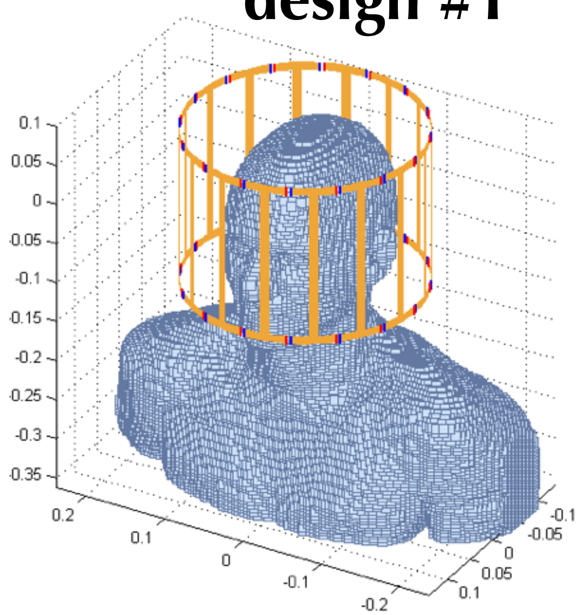


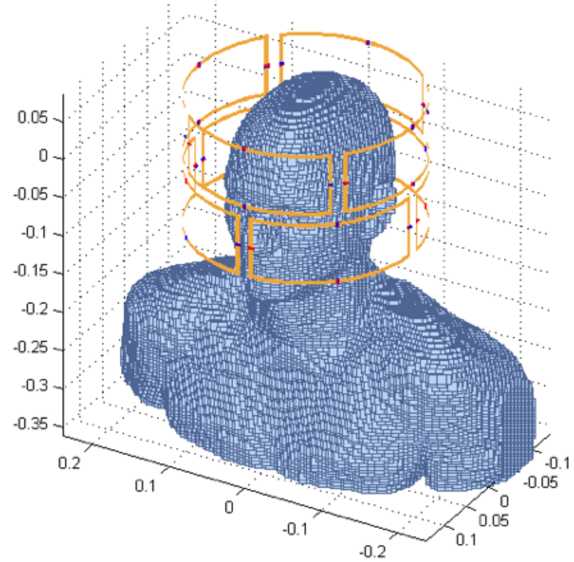
Impact coil design

- **Best out of 500 coil designs?** **Need full EM analysis of each design**

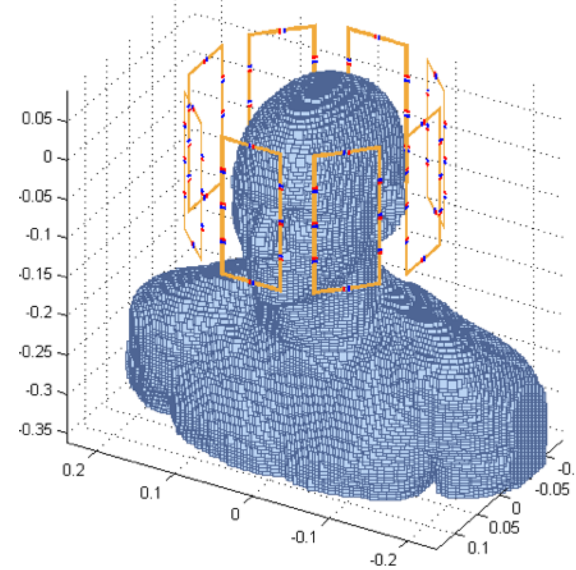
design #1



design #2



design #3



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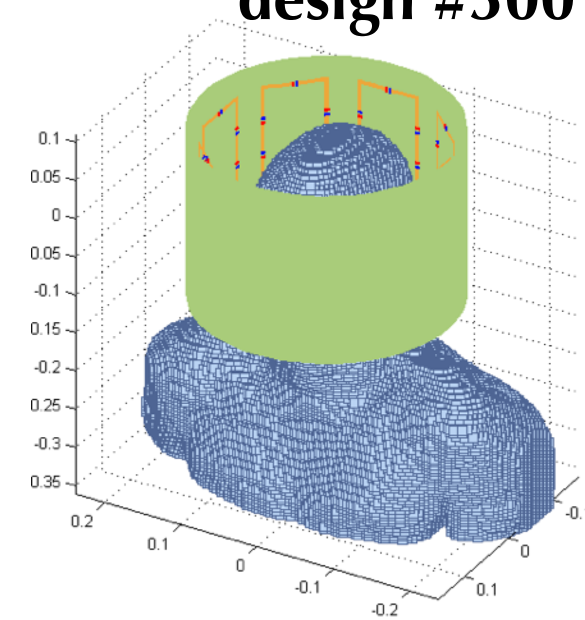
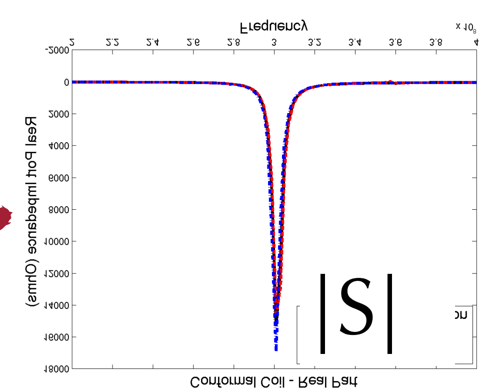
design #500

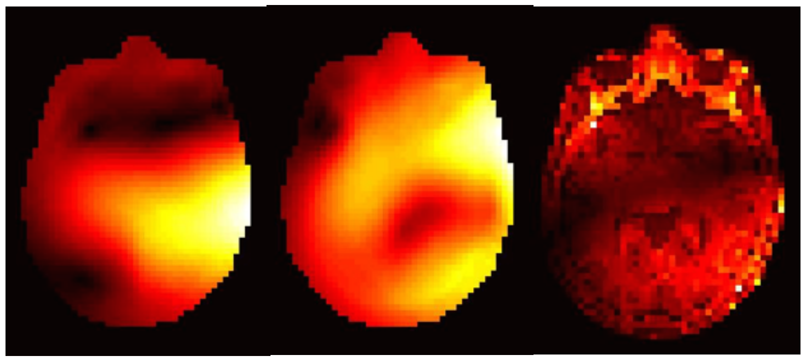


Existing tools:
> 1 month
(hours / design)

Full EM analysis



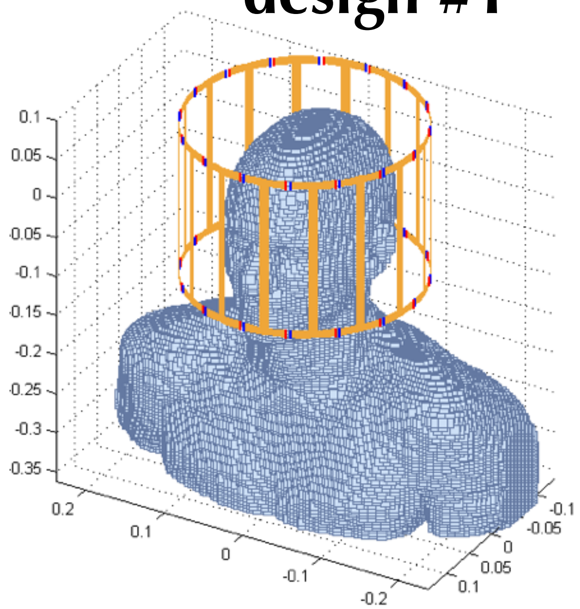
&



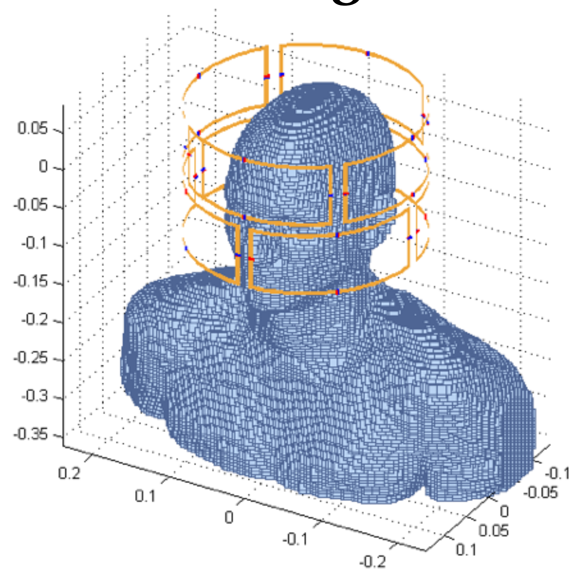
Impact coil design: allow design exploration

- **Best out of 500 coil designs? Need full EM analysis of each design**

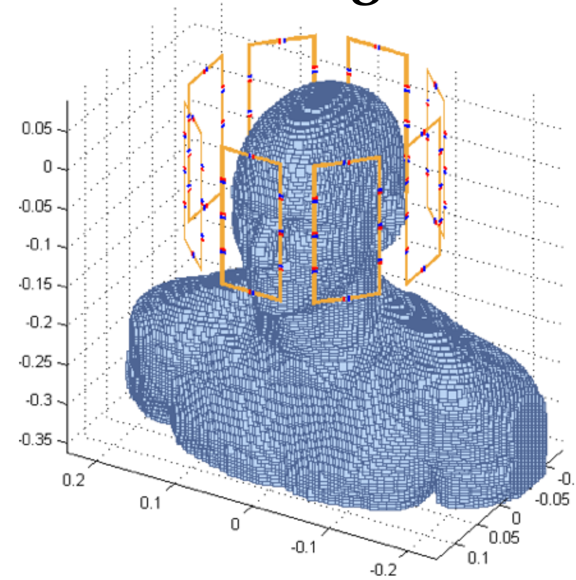
design #1



design #2

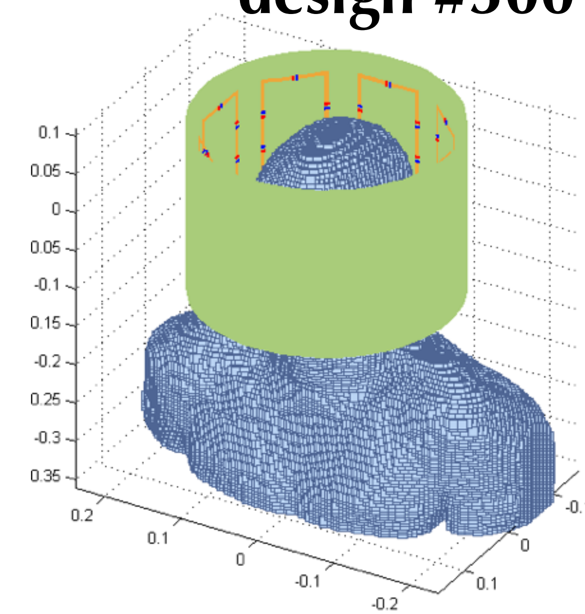


design #3



...

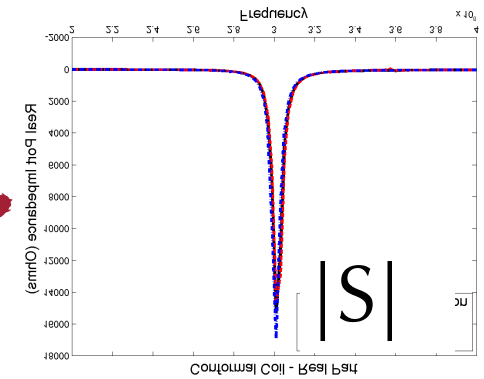
design #500



~~Existing tools:~~
~~> 1 month~~
~~(hours / design)~~

Proposed tool:
over the weekend
(~4min / design)

Full EM analysis



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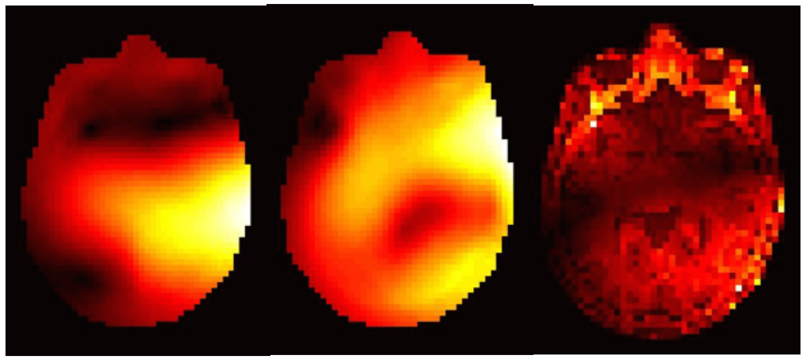
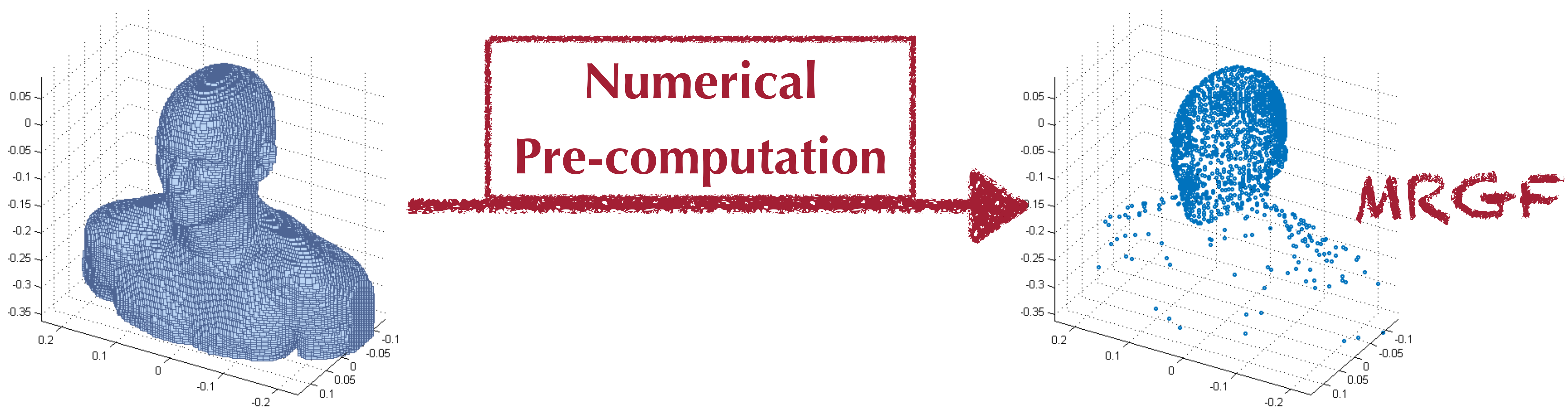


How?

- **MRI customized simulation tools**
  - based on **Integral Equation** methods
  - and **pre-computed Magnetic Resonance Green functions**



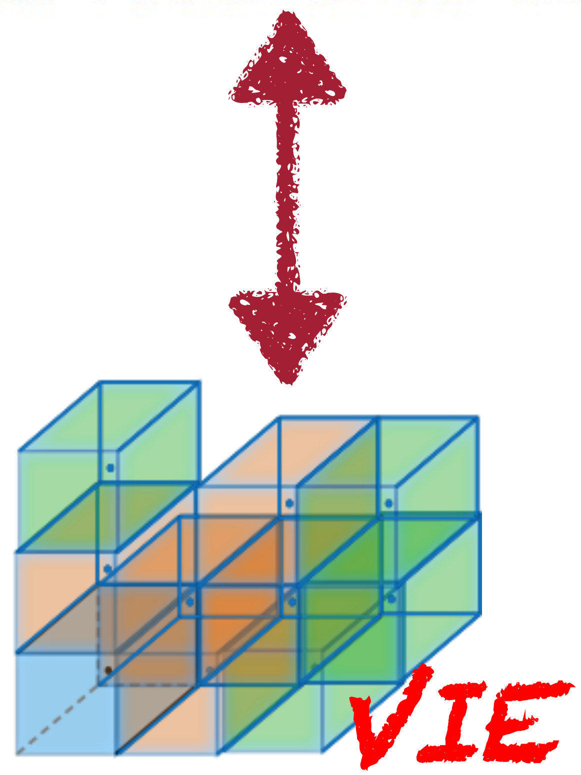
But first let's start from the basics

Magnetic Resonance specific Integral Equation suite

Core engine

- **Fast MR-specific Volume Integral Equation (VIE) solver**

Marie
magnetic resonance integral equation



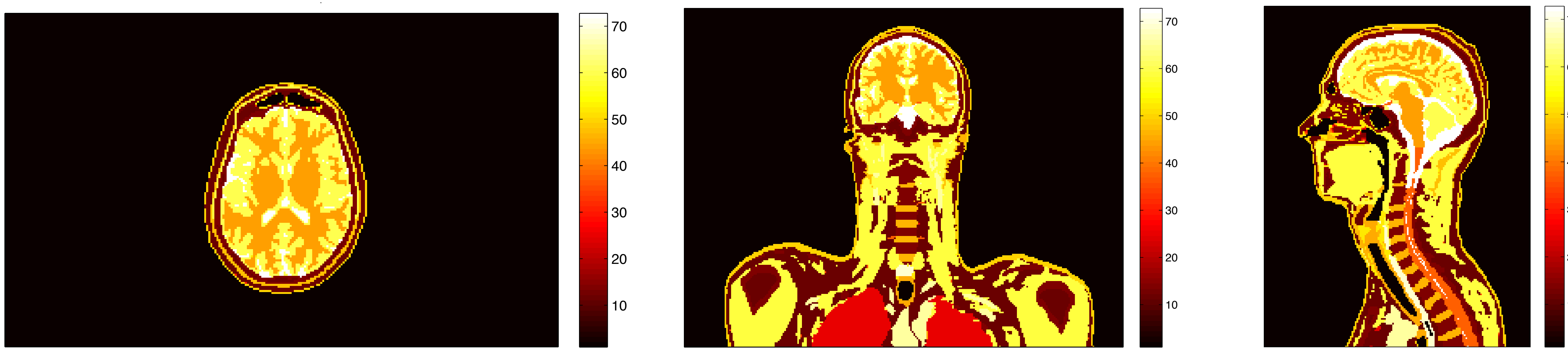
[4]

[4] **Polimeridis et al.**, "Stable FFT-JVIE solvers for fast analysis of highly inhomogeneous dielectric objects", **JCP 2014**

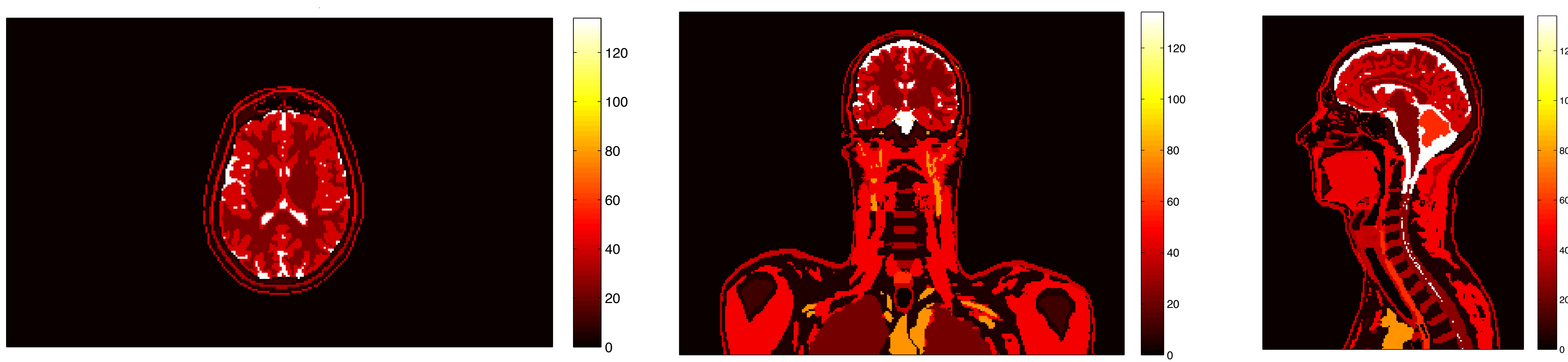
Challenges

- **Electric properties at 7T (298MHz)**

$\mathrm{Re}\{\epsilon_r\}$



$\mathrm{Im}\{\epsilon_r\}$

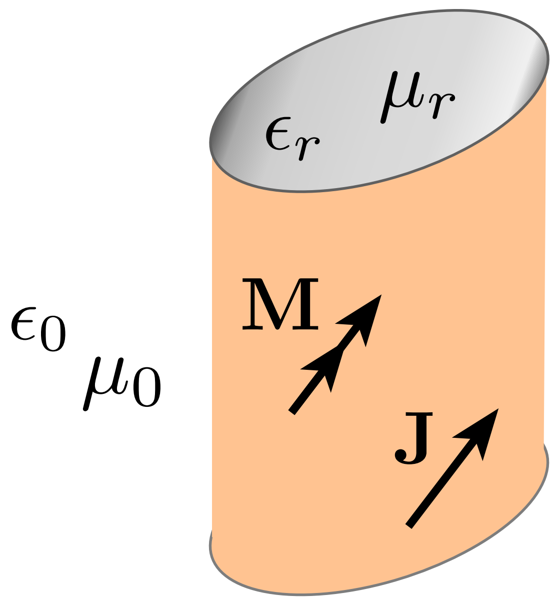


VIE solver: key facts

- **IE-based**
  - frequency domain (MRI is single frequency analysis)
  - reduces dimensionality and satisfy radiation conditions
  - easy to “couple” with other solvers
- **New current-based formulation (JVIE)**
  - natural formulation for MRI applications
- **Machine precision integration**
  - DEMCEM and DIRECTFN packages (http://web.mit.edu/thanos_p/www/)
- **FFT-based fast solver**
  - exploit voxel based data from MRI
- **Well conditioned system - fast convergence**
  - even for high contrast

VIE solver: formulation

- **Formulas for total fields**



$$\mathbf{e} = \mathbf{e}^{\mathrm{inc}} + \mathbf{e}^{\mathrm{sca}} = \mathbf{e}^{\mathrm{inc}} + \frac{1}{c_\epsilon}\mathcal{L}\mathbf{j} - \mathcal{K}\mathbf{m}$$

$$\mathbf{h} = \mathbf{h}^{\mathrm{inc}} + \mathbf{h}^{\mathrm{sca}} = \mathbf{h}^{\mathrm{inc}} + \frac{1}{c_\mu}\mathcal{L}\mathbf{m} + \mathcal{K}\mathbf{j}$$

$$\mathcal{L}\mathbf{u} \triangleq (k_0^2 + \nabla\nabla\cdot)\mathcal{S}\left(\mathbf{u};\Omega\right)(\mathbf{r})$$

$$\mathcal{K}\mathbf{u} \triangleq \nabla \times \mathcal{S}\left(\mathbf{u};\Omega\right)(\mathbf{r})$$

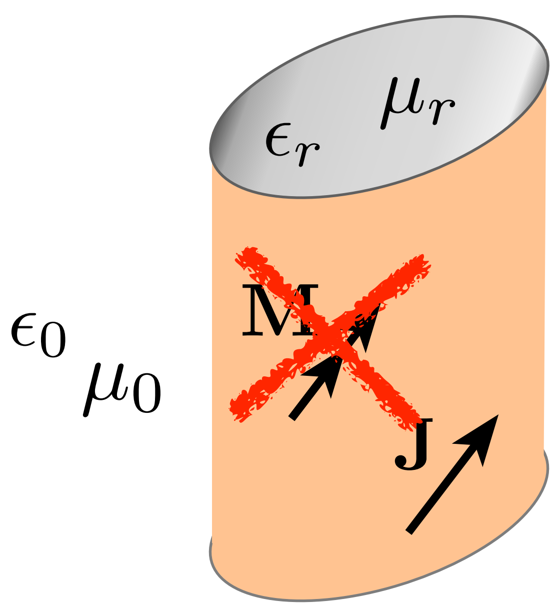
$$\mathcal{S}\left(\mathbf{u};\Omega\right)(\mathbf{r}) \triangleq \int_\Omega G(\mathbf{R})\mathbf{u}(\mathbf{r}')d\mathbf{r}'$$

$$\mathbf{j}(\mathbf{r}) \triangleq c_\epsilon \chi_\epsilon(\mathbf{r})\,\mathbf{e}(\mathbf{r})$$

$$c_\epsilon = j\omega\epsilon_0$$
$$\chi_\epsilon = \epsilon_r(\mathbf{r}) - 1$$

VIE solver: formulation

- **Formulas for total fields**
  - non-magnetic material



$$\mathbf{e} = \mathbf{e}^{\mathrm{inc}} + \mathbf{e}^{\mathrm{sca}} = \mathbf{e}^{\mathrm{inc}} + \frac{1}{c_\epsilon}\mathcal{L}\mathbf{j} - \cancel{\mathcal{K}\mathbf{m}}$$

$$\mathbf{h} = \mathbf{h}^{\mathrm{inc}} + \mathbf{h}^{\mathrm{sca}} = \mathbf{h}^{\mathrm{inc}} + \cancel{\frac{1}{c_\mu}\mathcal{L}\mathbf{m}} + \mathcal{K}\mathbf{j}$$

$$\mathbf{j}(\mathbf{r}) \triangleq c_\epsilon \chi_\epsilon(\mathbf{r})\,\mathbf{e}(\mathbf{r})$$

$$c_\epsilon = j\omega\epsilon_0$$
$$\chi_\epsilon = \epsilon_r(\mathbf{r}) - 1$$

$$\mathcal{L}\mathbf{u} \triangleq (k_0^2 + \nabla\nabla\cdot)\mathcal{S}\left(\mathbf{u};\Omega\right)(\mathbf{r})$$
$$\mathcal{K}\mathbf{u} \triangleq \nabla\times\mathcal{S}\left(\mathbf{u};\Omega\right)(\mathbf{r})$$

$$\mathcal{S}\left(\mathbf{u};\Omega\right)(\mathbf{r}) \triangleq \int_\Omega G(\mathbf{R})\mathbf{u}(\mathbf{r}')d\mathbf{r}'$$

VIE solver: formulation

- **Select a current based formulation**
  - behaves well for high contrast

$$\lim_{\epsilon_r \to \infty} \text{JVIE}: \qquad (\mathcal{I} - \mathcal{N})\mathbf{j} = c_\epsilon\, \mathbf{e}^{\text{inc}}$$

  - Two possible formulations
    - Second one is naturally pre-conditioned

$$\mathbf{JVIE_I}: \qquad (\mathcal{M}_{\epsilon_r} - \mathcal{M}_{\chi_\epsilon}\mathcal{N})\,\mathbf{j} = c_\epsilon \mathcal{M}_{\chi_\epsilon}\,\mathbf{e}^{\text{inc}}$$

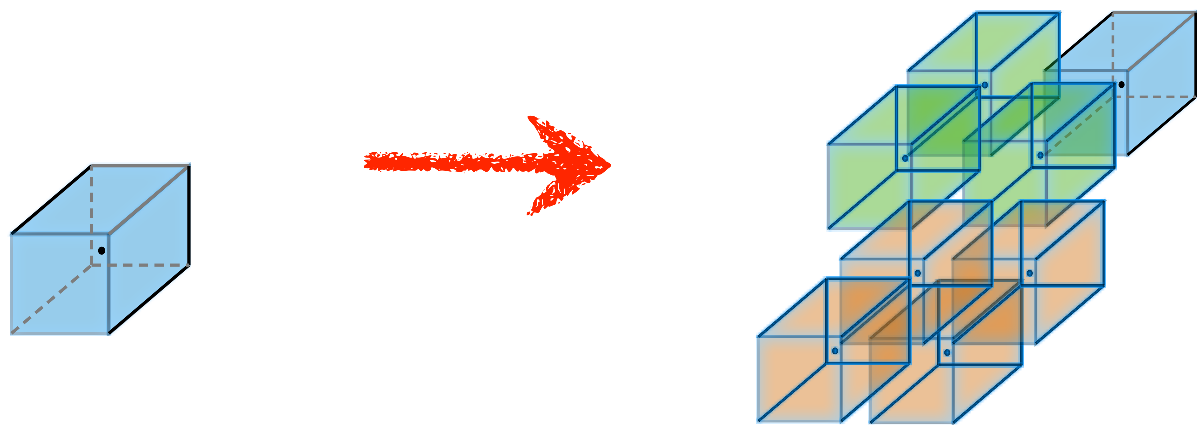
$$\mathbf{JVIE_{II}}: \qquad (\mathcal{I} - \mathcal{M}_{\tau_\epsilon}\mathcal{N})\,\mathbf{j} = c_\epsilon \mathcal{M}_{\tau_\epsilon}\,\mathbf{e}^{\text{inc}}$$

$$\tau_\epsilon = \chi_\epsilon / \epsilon_r$$

VIE solver: numerics

- **Voxel as support**
  - natural discretization of MRI applications
  - transform volume integrals into surface
  - allows to apply FFT based approaches

Toeplitz → *Circulant*



$Ax = b$ → *Iterative Solver* → *M-v product* → *FFT* $N \log(N)$

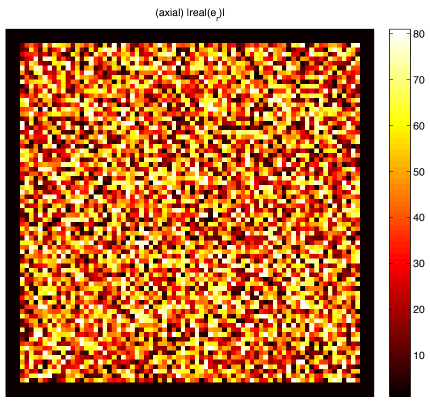
VIE solver: convergence

- **Extremely challenging case**

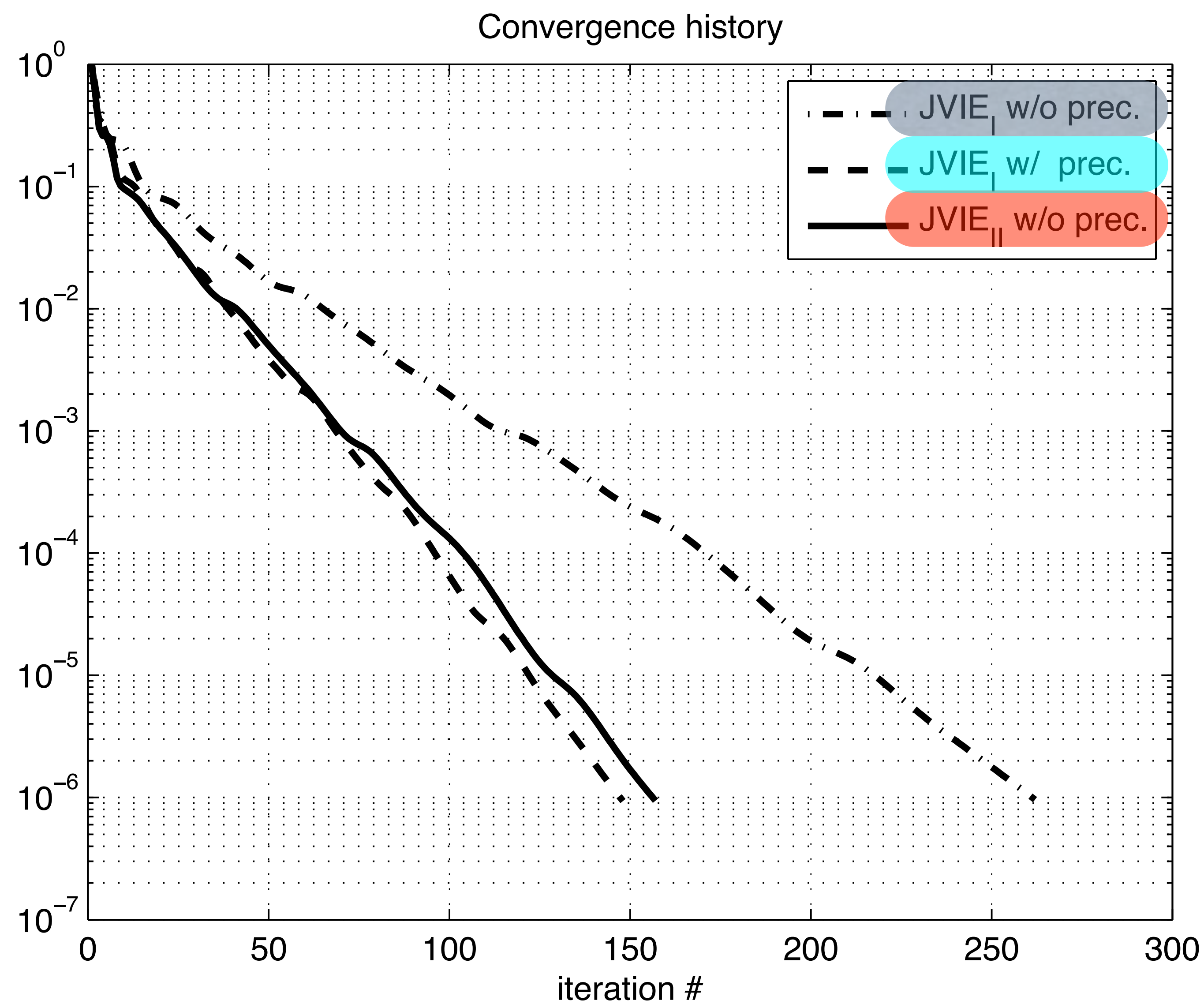
Cube

$kL = 1$

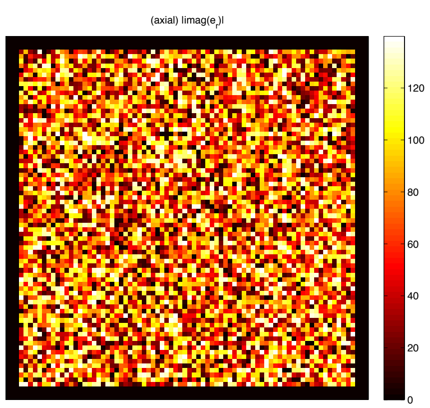
$1 \leq \mathrm{Re}\{\epsilon_r\} \leq 80$



GMRES

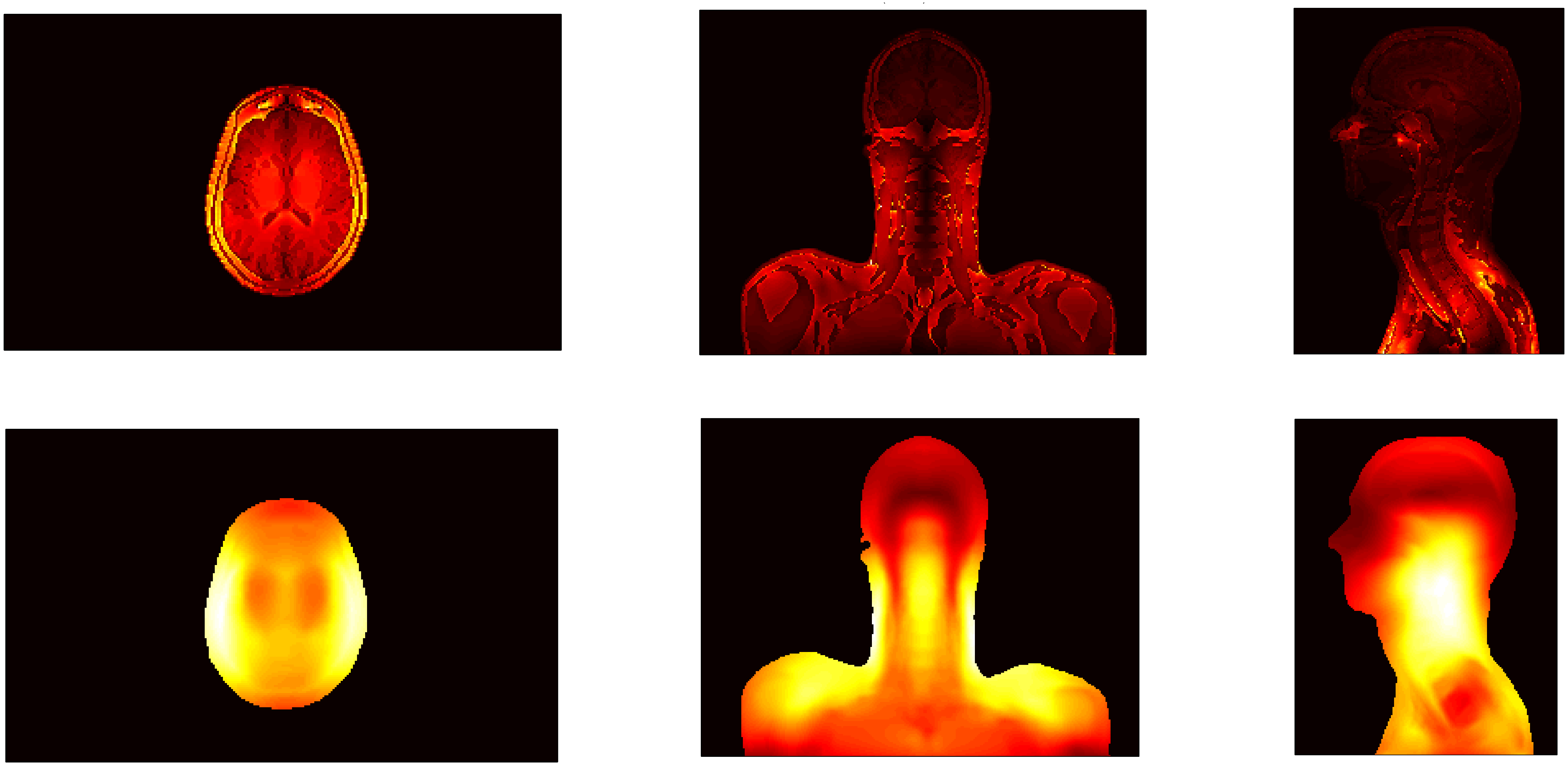


$0 \leq |\mathrm{Im}\{\epsilon_r\}| \leq 140$



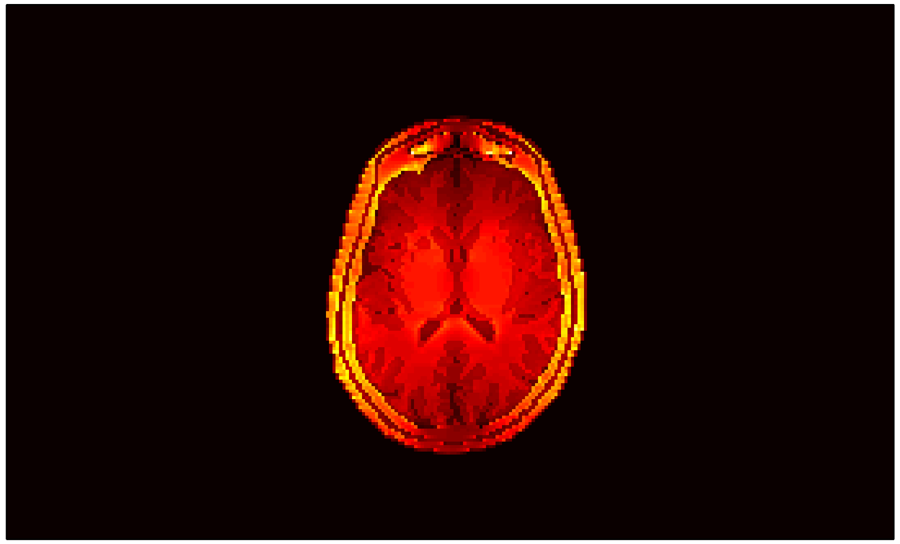
VIE solver: performance

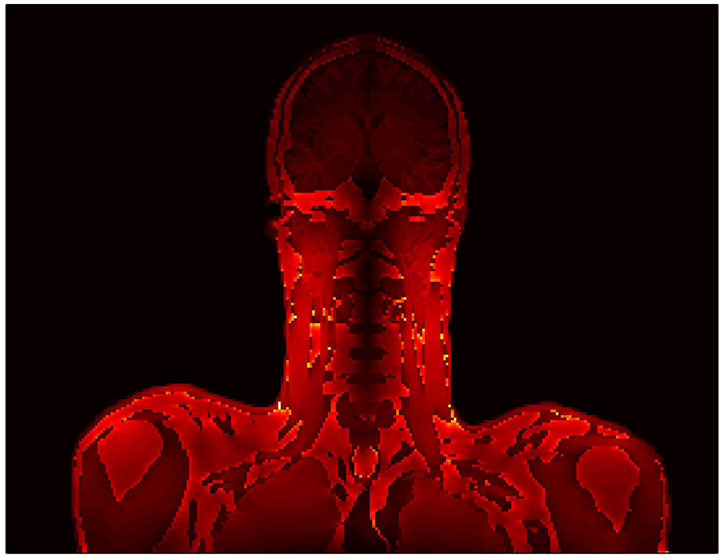
- **Realistic human body model at 7T**

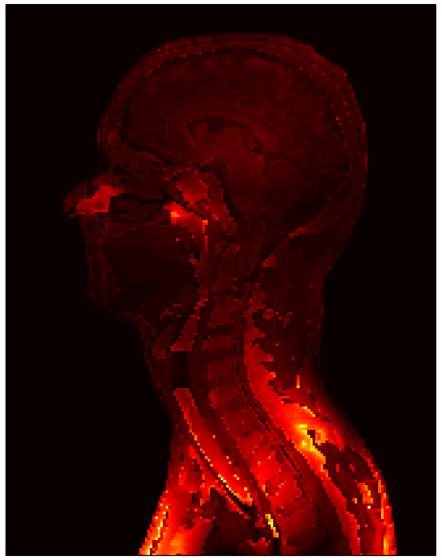


VIE solver: performance

- **Realistic human body model at 7T**







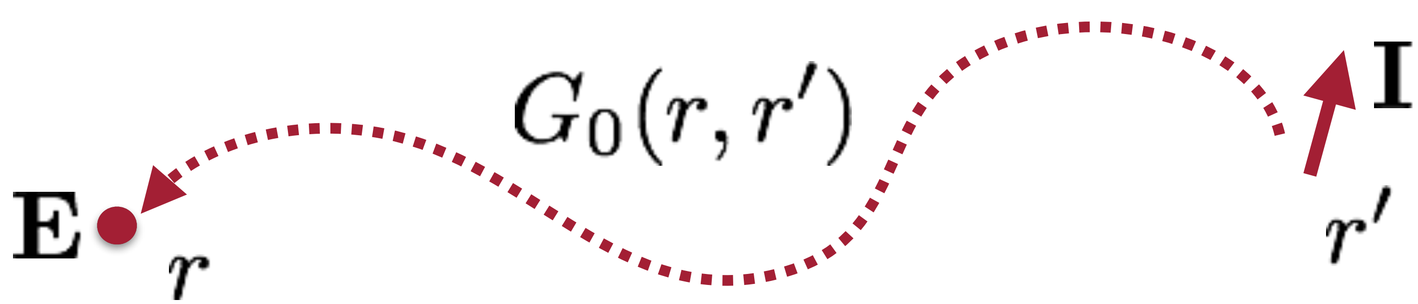
		OFFLINE	GMRES	GMRES (40)	GMRES (40,5)	BICG	BICGSTAB	QMR	TFQMR
5mm	Serial Parallel Speed-Up	20 s 5 s 4×	15 s 7 s 2.1×	15 s 5 s (3 s) 3.0× (5×)	13 s 5 s 2.6×	28 s 4 s 7.0×	16 s 3 s 5.3×	23 s 4 s 5.7×	17s 3 s 5.6×
2.5mm	Serial Parallel Speed-Up	146 s 27 s 5.4×	146 s 65 s 2.2×	142 s 48 s (23 s) 2.9× (6.1×)	125 s 42 s 2.9×	276 s 40 s 6.9×	162 s 25 s 6.4×	266 s 40 s 6.6×	174 s 32 s 5.4×

3,000,000 unknowns!

Integral Equation solvers for MRI coil analysis

What is a Green Function?

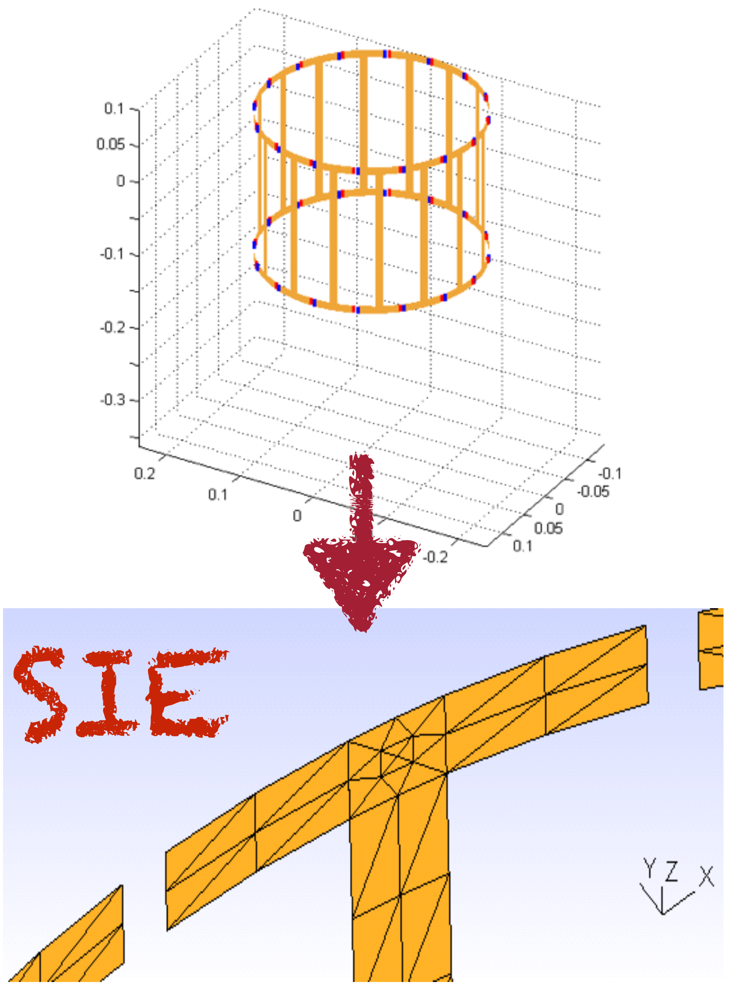
- **Green function [2]**
  - gives fundamental solution of the problem at any point
  - satisfies (by definition) radiation conditions (no ABC or PML)



[2] **Tai**, “Dyadic Green functions in electromagnetic theory”. 1994

Use Green function to model coils

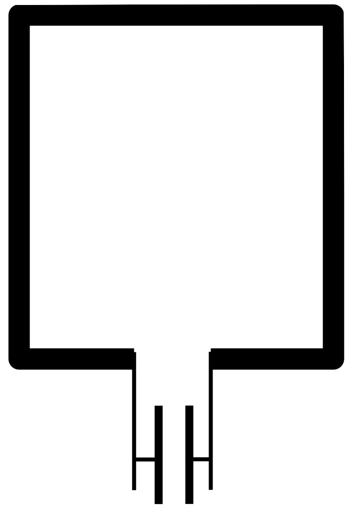
- **Green function**
  - gives fundamental solution of the problem at any point
  - satisfies (by definition) radiation conditions (no ABC or PML)
- **At the core of Integral Equation methods**

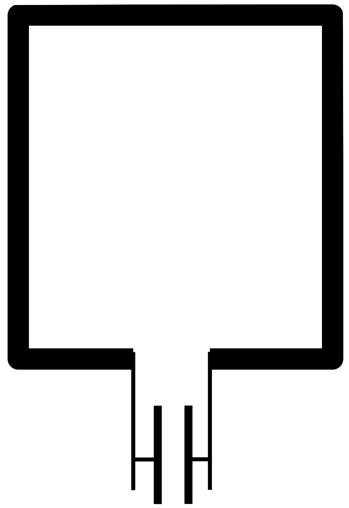
[3] 

[3] **Rao et al.**, "Electromagnetic scattering by surfaces of arbitrary shape", **TAP 1982**

Integral equations to model coils

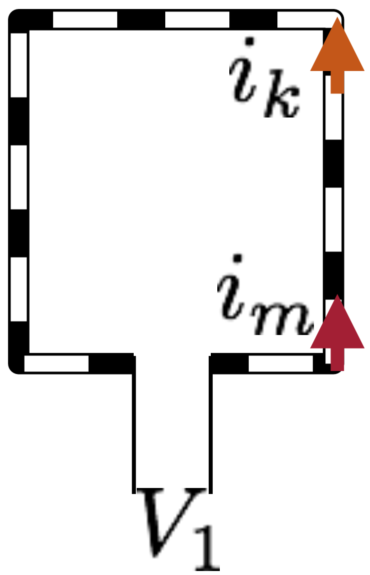
- **Two homogeneous coils**
  - free space

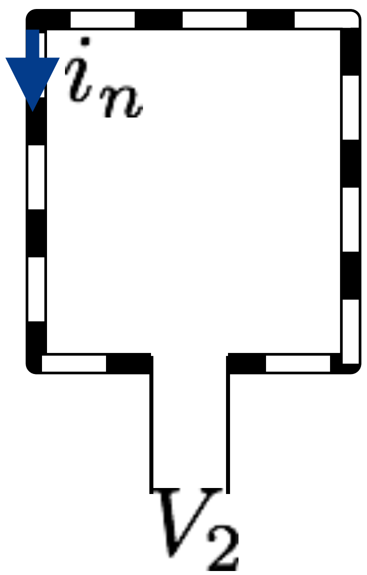




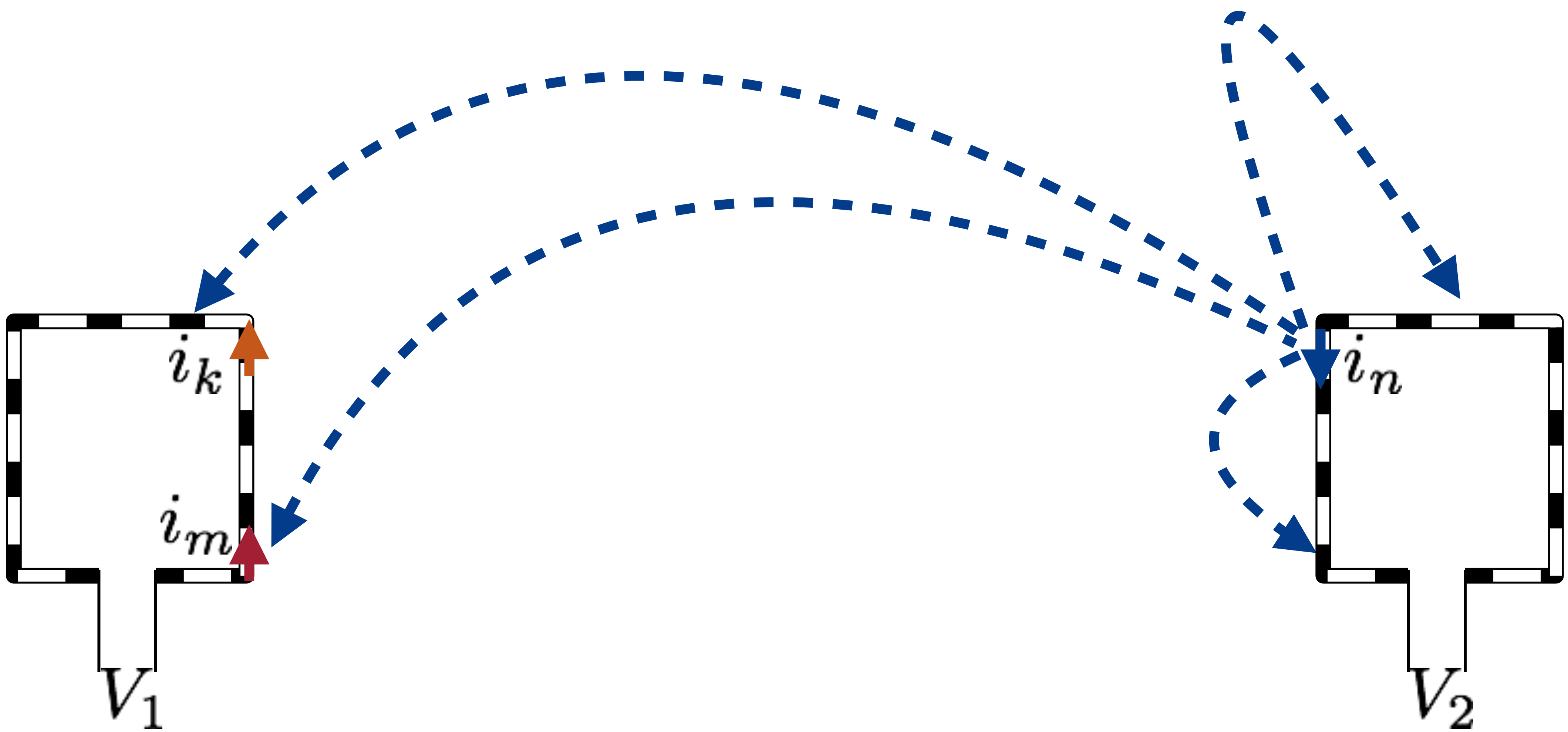
Integral equations to model coils

- **Discretize the conductors (not the air)**
  - connection segments define a port
  - constant current at each element





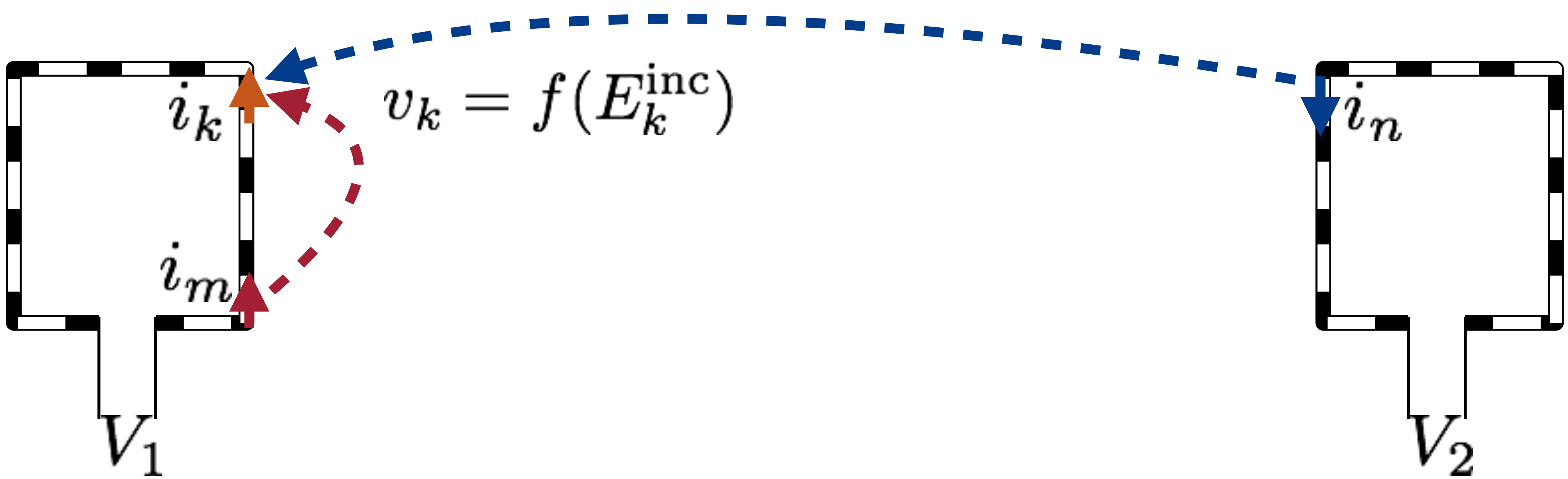
- **Discretize the conductors (not the air)**
  - connection segments define a port
  - constant current at each element
  - each current radiates a field



Integral equations to model coils

- **Discretize the conductors (not the air)**
  - connection segments define a port
  - constant current at each element
  - each current radiates a field
  - field at each element:
    - contribution from all currents

$$\ldots + \underline{G(k,m)}i_m + \underline{G(k,k)}i_k + \underline{G(k,n)}i_n + \ldots = E_k^{\text{inc}}$$



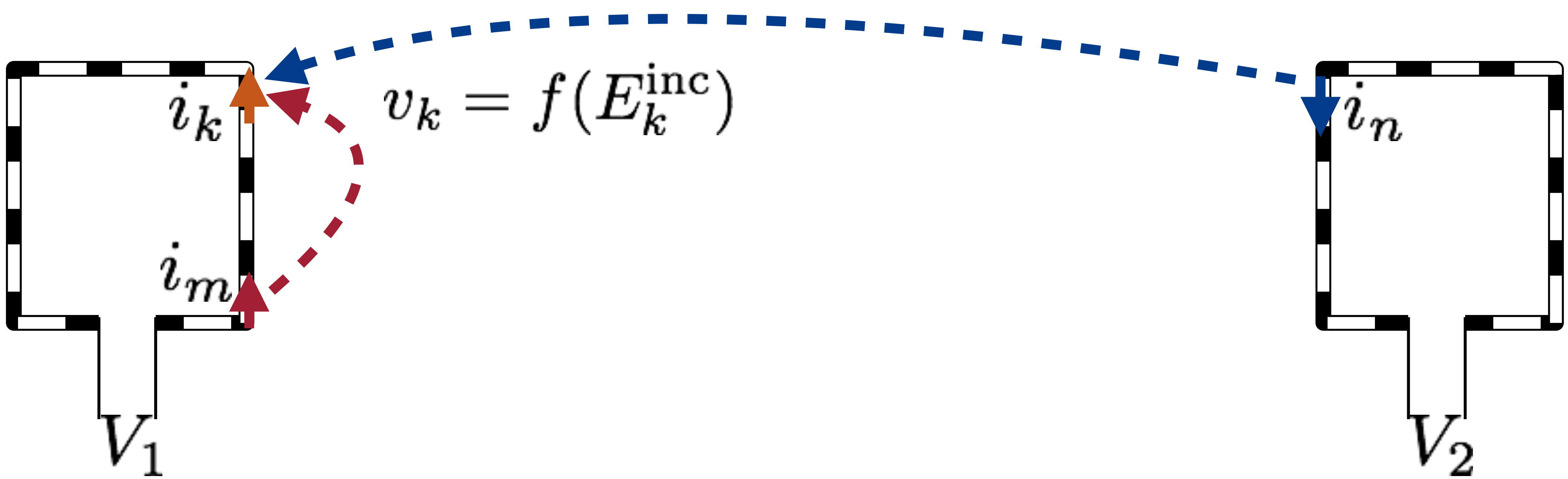
Integral equations to model coils

- **Construct system of equations**
  - relates the variables (currents)
  - external excitations (port voltage)

$$Z^{cc} I^c = V^c$$

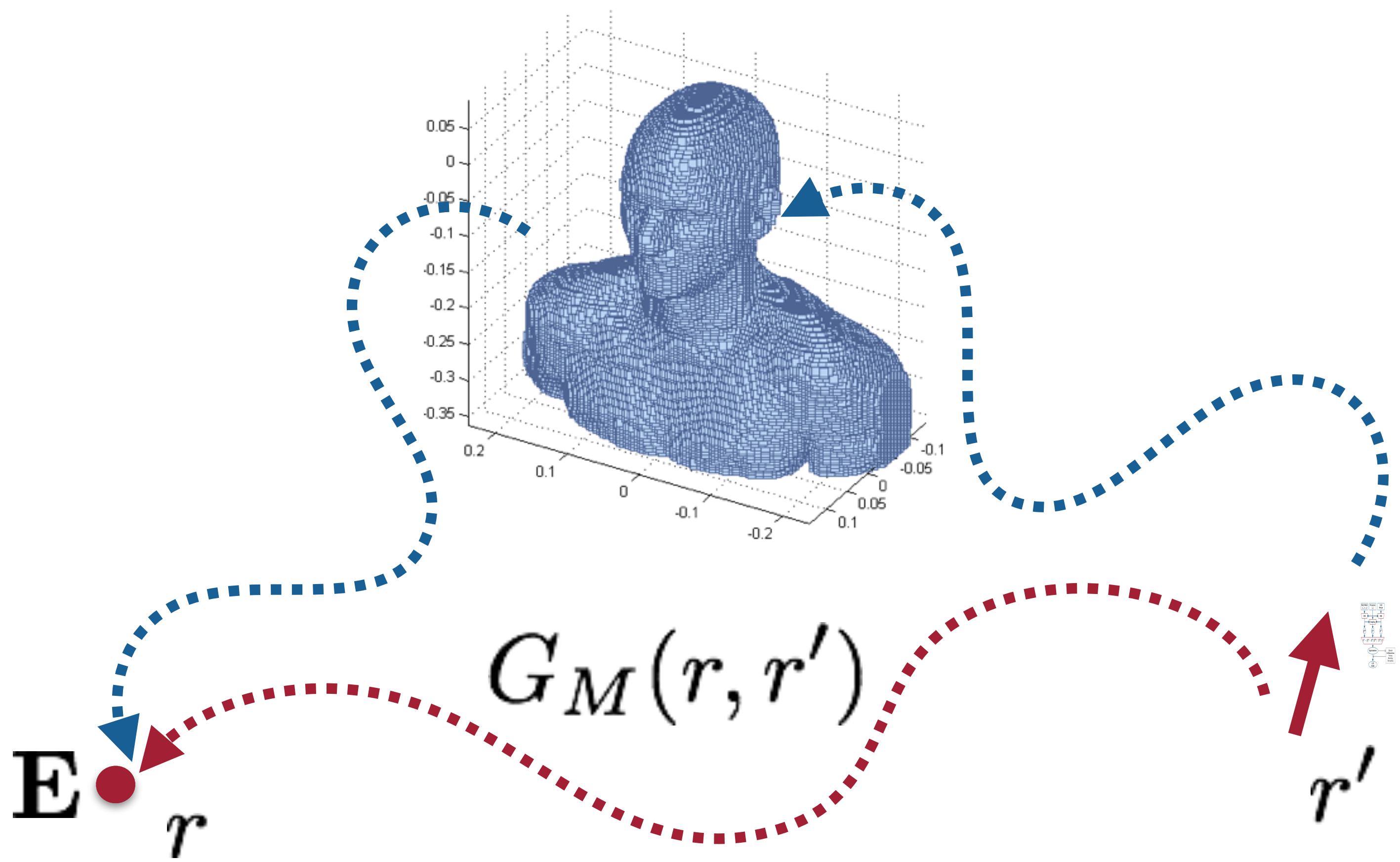
$$z_{k,m} \qquad z_{k,k} \qquad z_{k,n}$$

$$\ldots + \underline{G(k,m) i_m} + \underline{G(k,k) i_k} + \underline{G(k,n) i_n} + \ldots = E_k^{\text{inc}}$$



Now…if there is a scatterer?

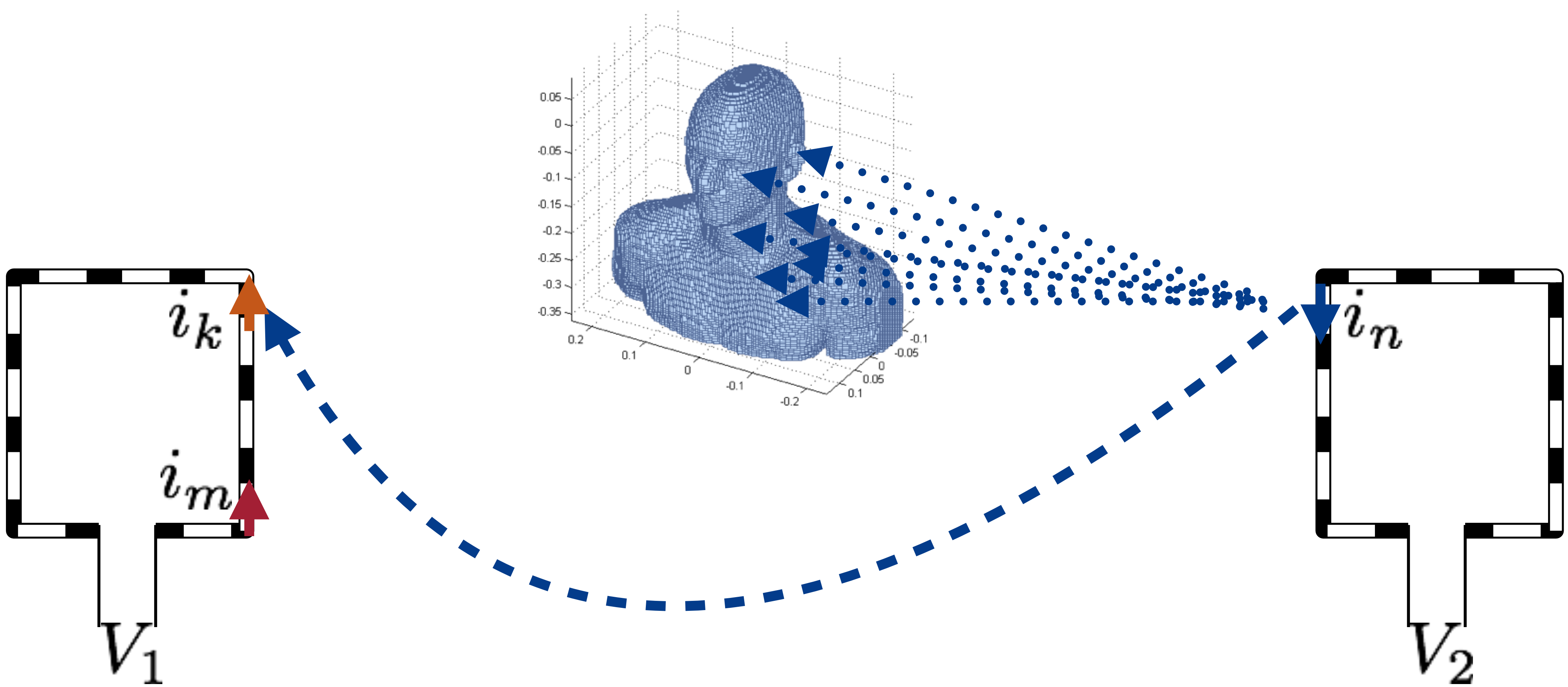
- **There is a perturbation induced by the body**



Coupled integral equation solver

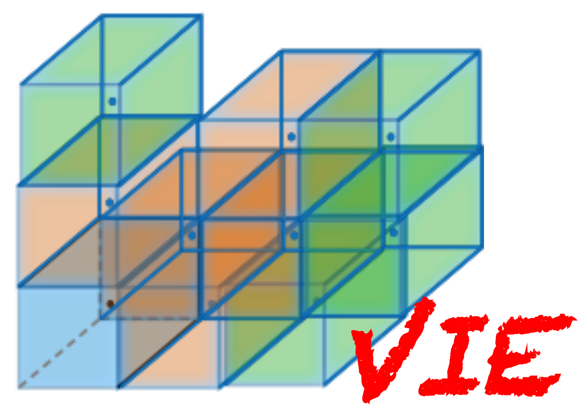
- **There is a perturbation induced by the body**

$$E^{\text{inc}} = G(r', r_n) i_n$$

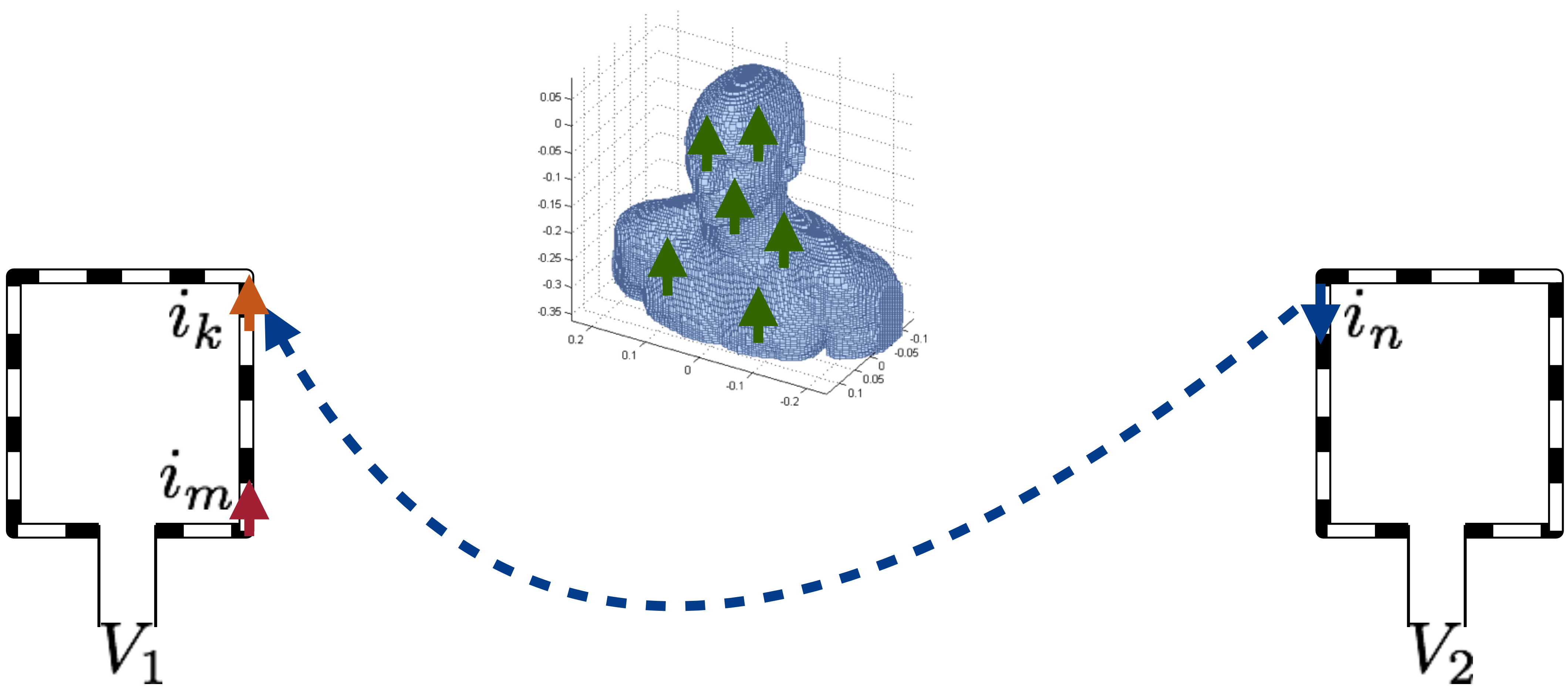


Coupled integral equation solver

- **There is a perturbation induced by the body**



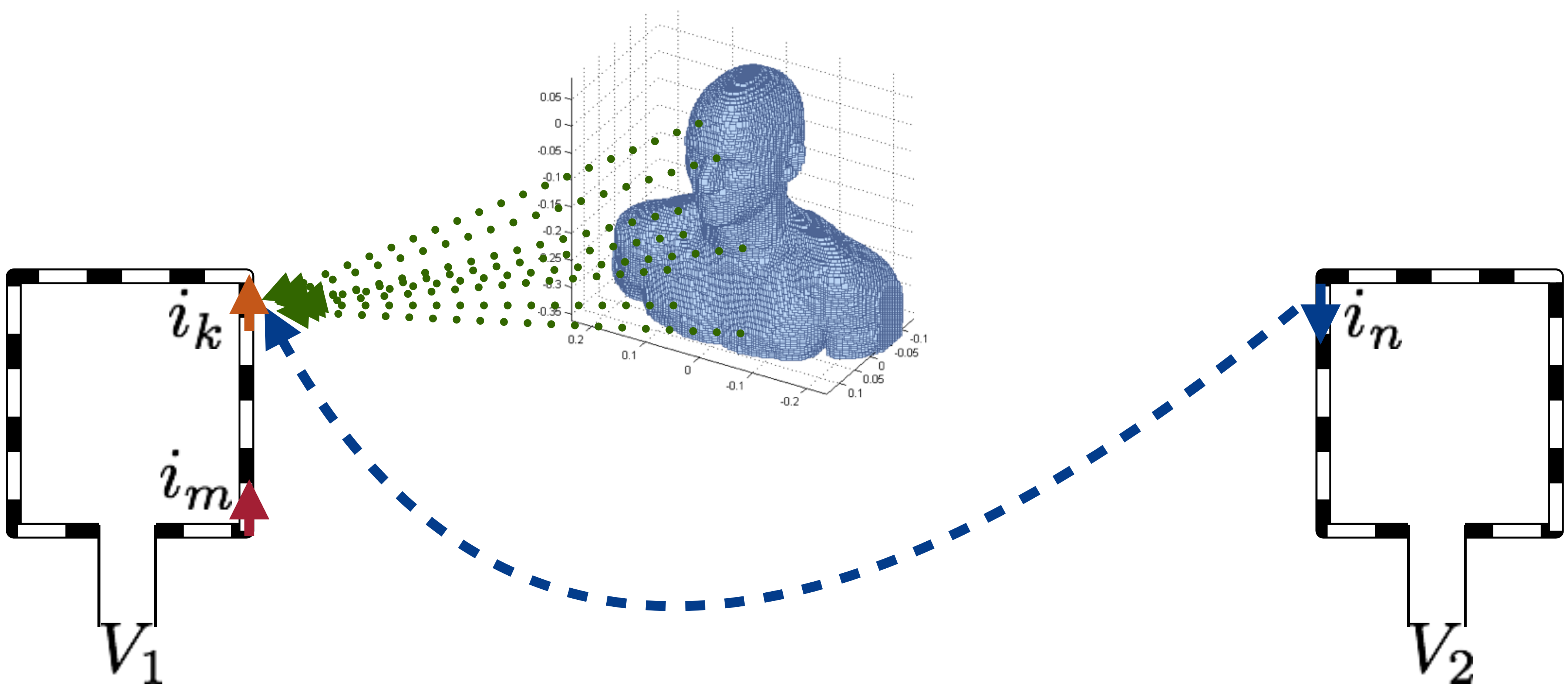
$$J^b = (Z^{bb})^{-1} E^{\text{inc}}$$



Coupled integral equation solver

- **There is a perturbation induced by the body**

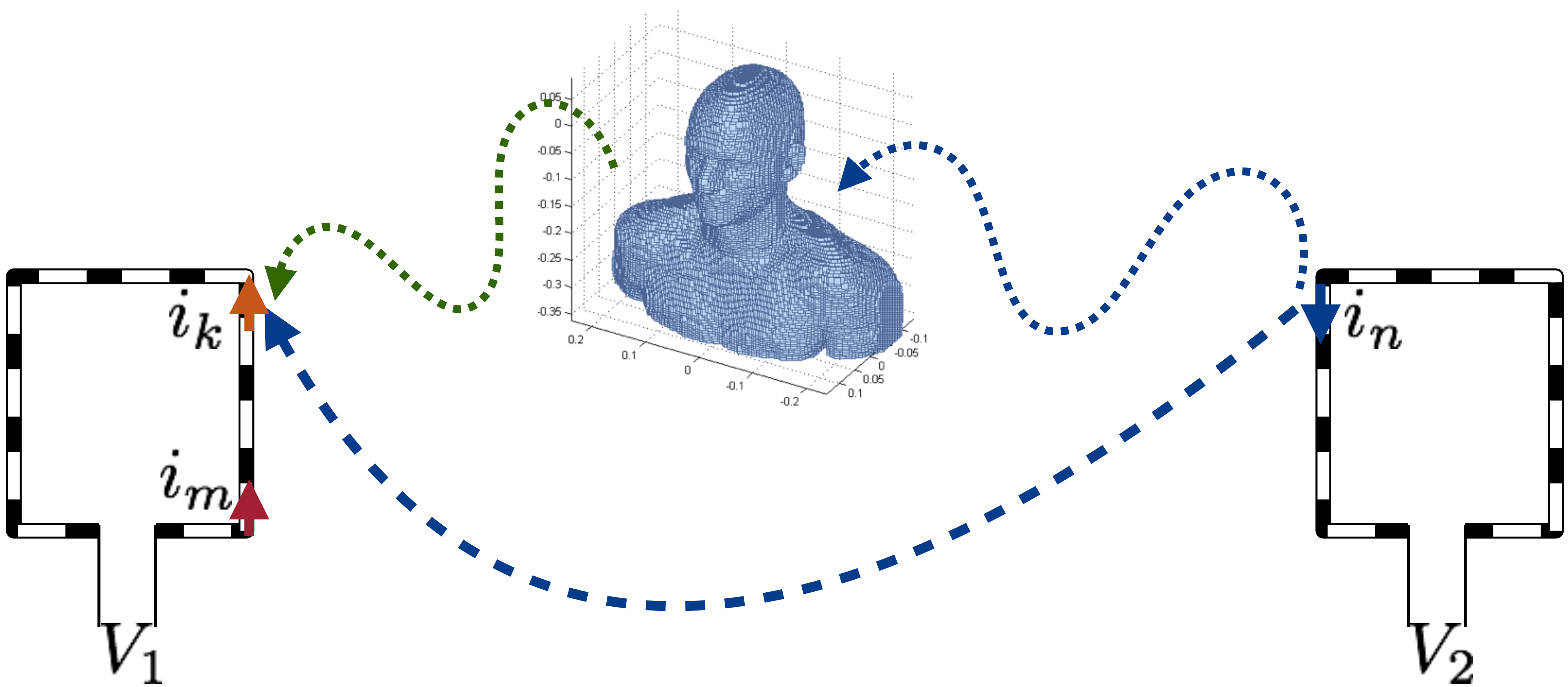
$$E^{\text{inc}} = G(r_k, r')J^b$$



Coupled integral equation solver

- **There is a perturbation induced by the body**
  - total contribution of elements is direct+scattered

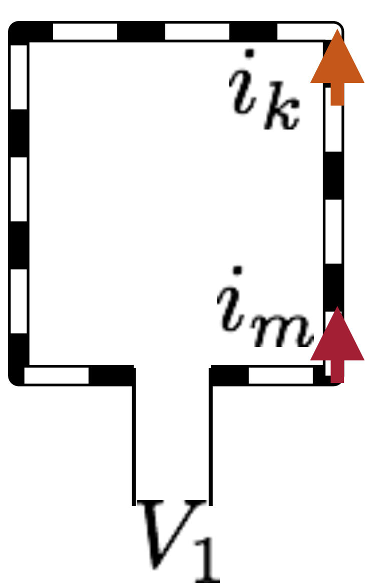
$$E_k^{\text{inc}} = G(k,n)i_n + G(k,r')(Z^{bb})^{-1}G(r',n)i_n$$

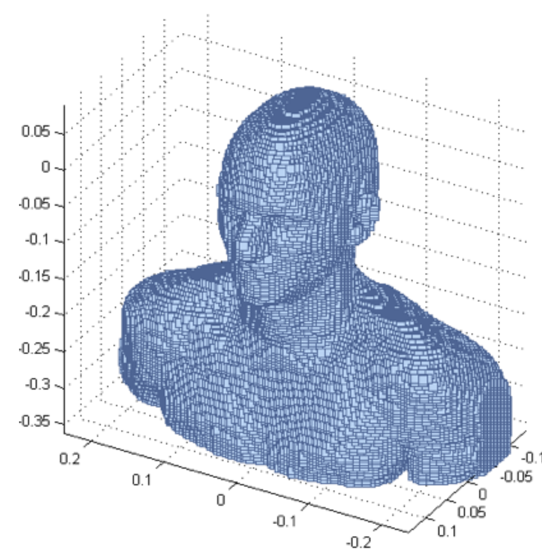


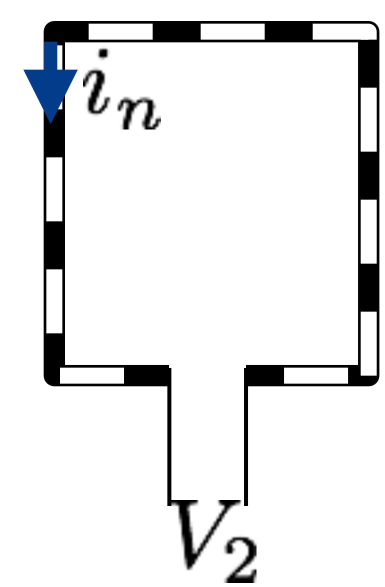
Coupled integral equation solver

- **There is a perturbation induced by the body**
  - total contribution of elements is direct+scattered
  - assemble total system

$$\left(Z^{cc} - Z^{cb}(Z^{bb})^{-1}Z^{cb}\right) I^c = V^c$$

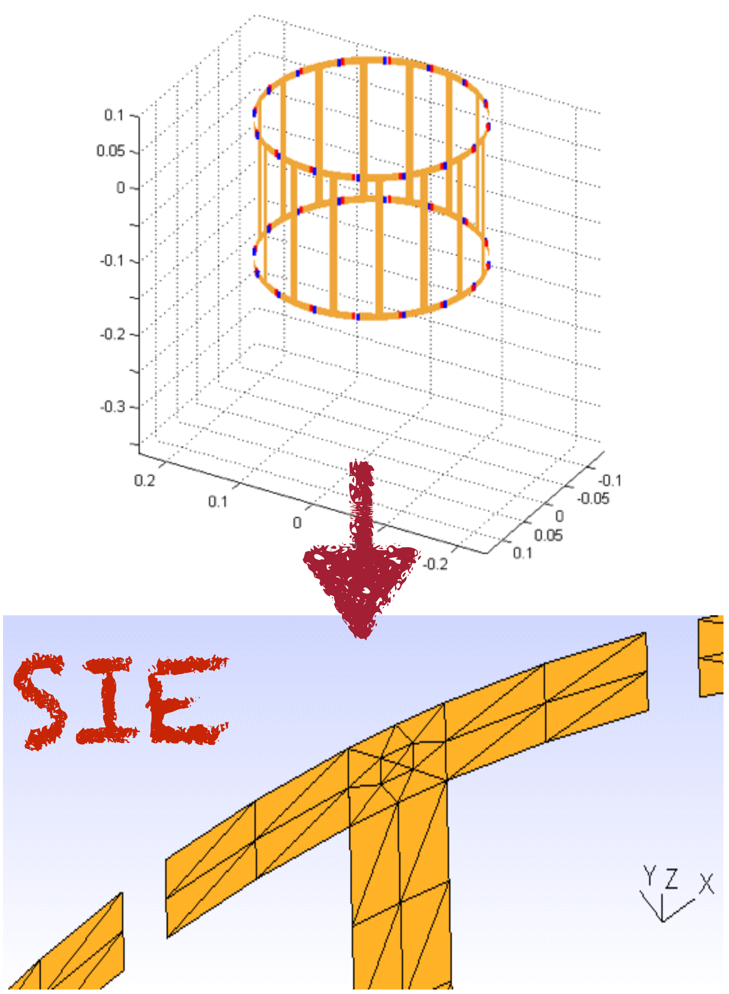


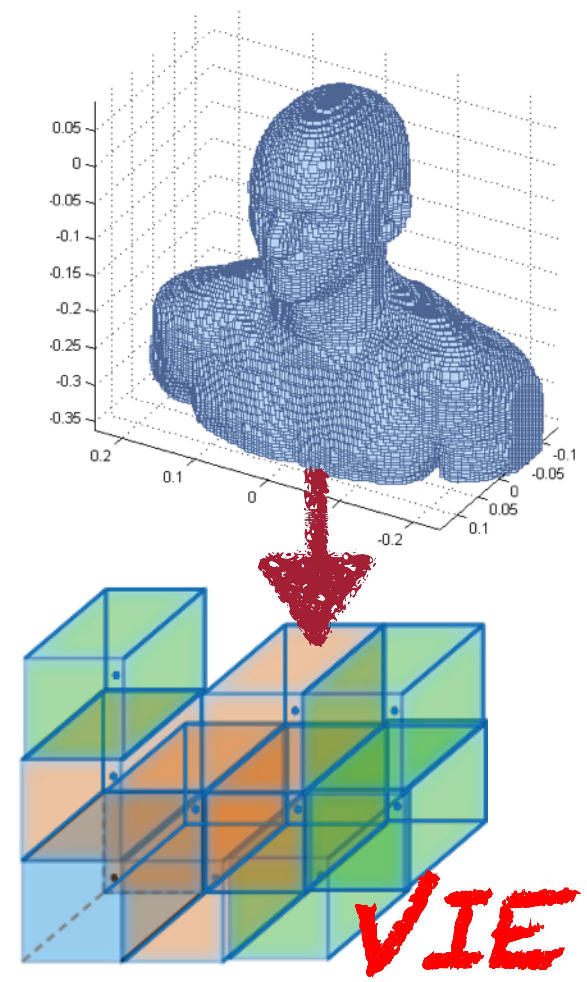




Couple surface and volume solvers

- **SIE for coils and VIE for body model**

[3] 

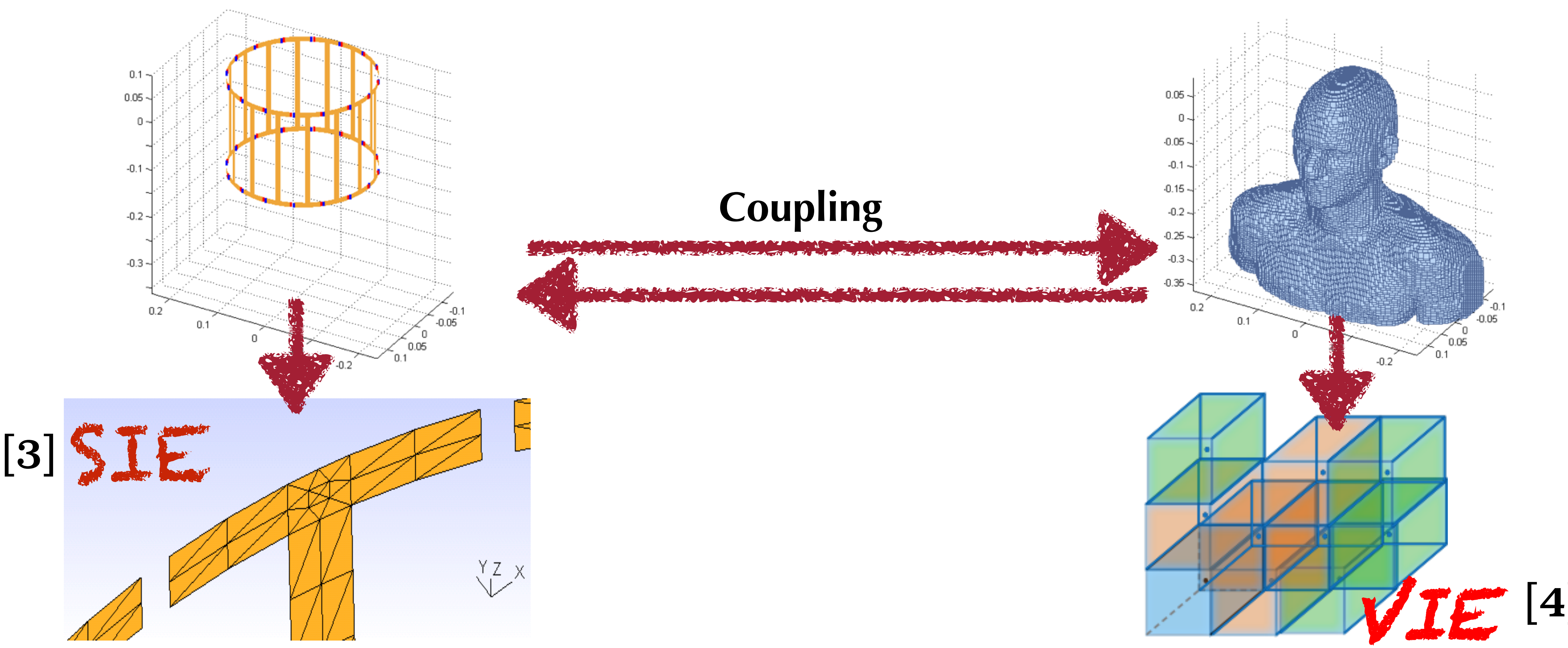


[4]

[3] **Rao et al.**, "Electromagnetic scattering by surfaces of arbitrary shape", **TAP 1982**
[4] **Polimeridis et al.**, "Stable FFT-JVIE solvers for fast analysis of highly inhomogeneous dielectric objects", **JCP 2014**

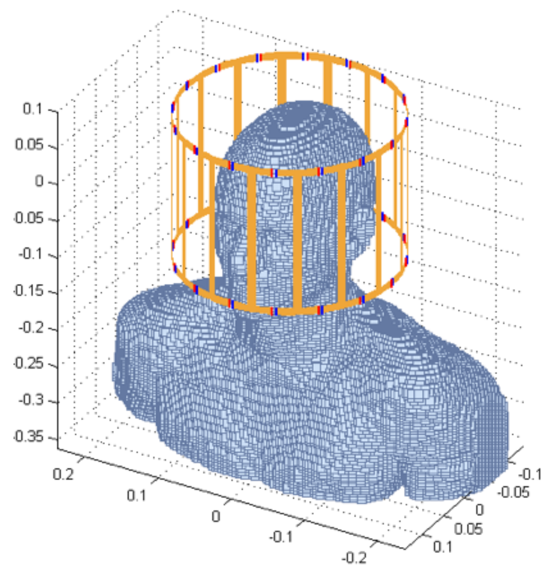
Couple surface and volume solvers

- **SIE for coils and VIE for body model**
  - Coupling is done via free-space Green function
  - Superposition + linearity: seamlessly combination

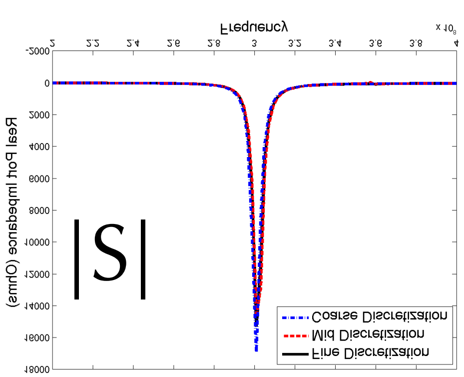


[3] **Rao et al.**, "Electromagnetic scattering by surfaces of arbitrary shape", **TAP 1982**
[4] **Polimeridis et al.**, "Stable FFT-JVIE solvers for fast analysis of highly inhomogeneous dielectric objects", **JCP 2014**

Combine into full-wave EM tool



Full-wave EM
Integral Equation Solver



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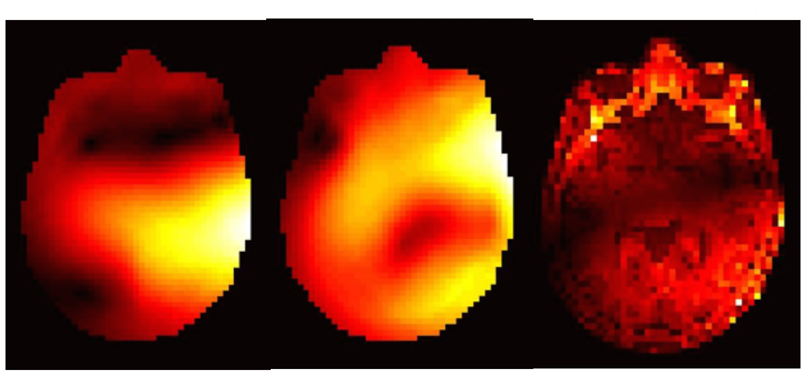
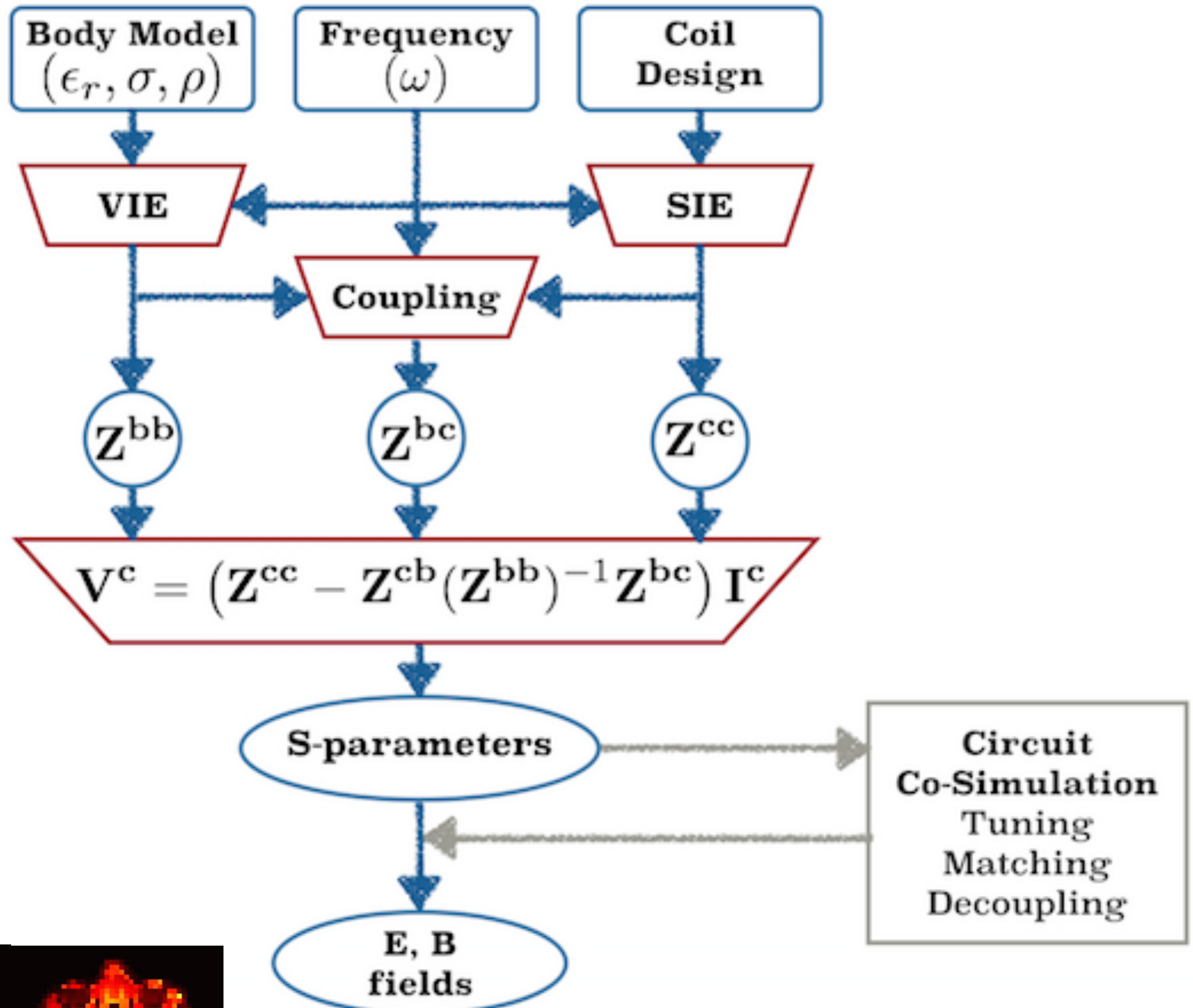




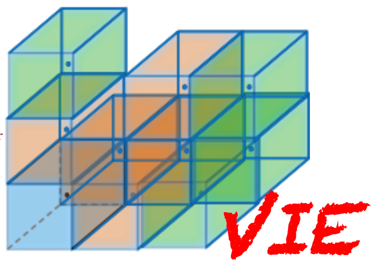
Iterative solver

- **Two level iterative solver** $\left(Z^{cc} - Z^{cb}(Z^{bb})^{-1}Z^{cb}\right) I^c = V^c$

$$\text{Guess} \quad I^c$$

$$E^{\text{inc}} = Z^{bc} I^c$$

$$\text{Solve} \quad Z^{bb} J^b = E^{\text{inc}} \quad \text{for} \quad J^b$$



$$x = Z^{cc} I^c - Z^{cb} J^b$$

$$r = V^c - x$$

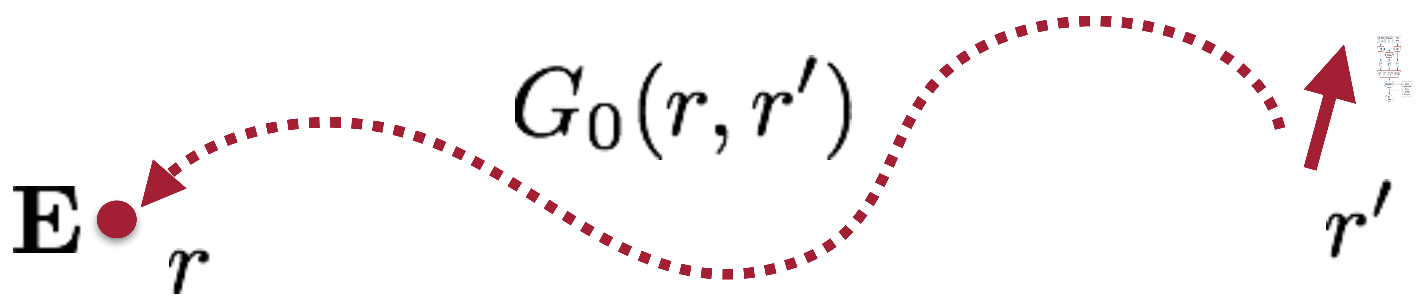
$$||r|| < \text{tol}$$

$$\text{return} \quad J^b \quad I^c$$

Basis for the EM fields in realistic body models

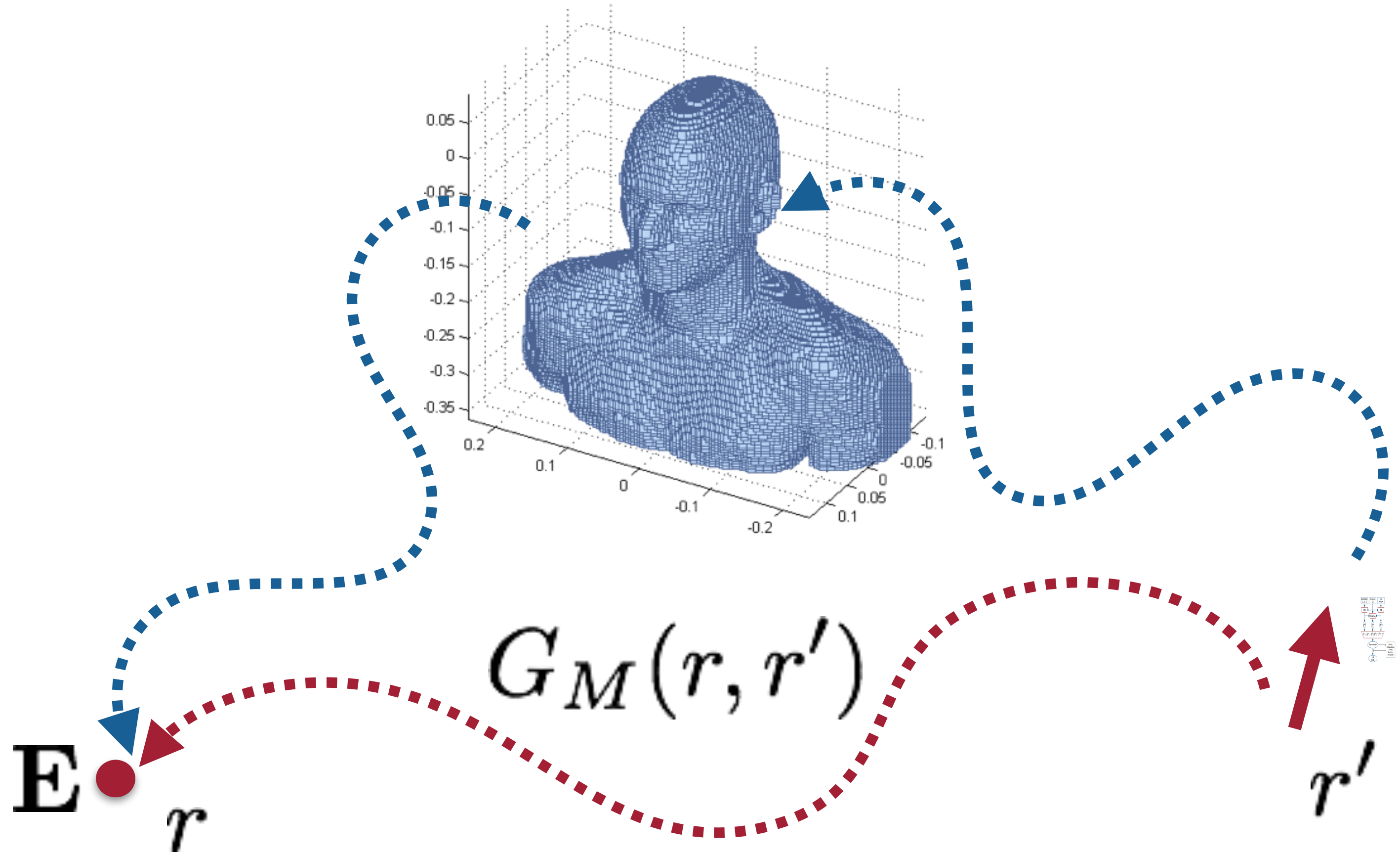
Now…a Magnetic Resonance Green Function?

- **Free space Green function**
  - fundamental solution of the problem at any point



Now…a Magnetic Resonance Green Function?

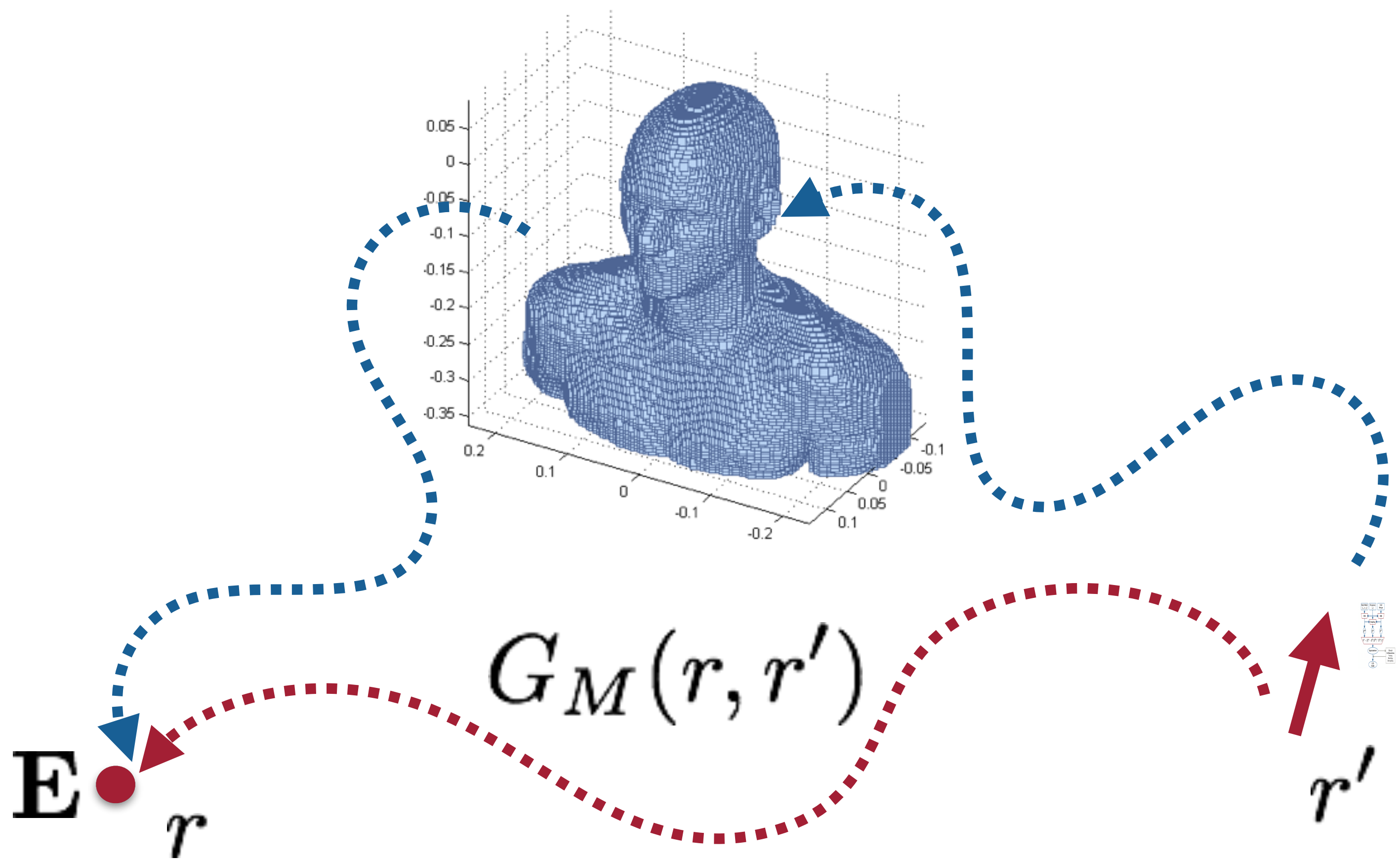
- **Magnetic Resonance Green function**
  - fundamental solution of body scattering problem at any point



- **Realistic human body model: too complex**
  - How to get an analytical function?

Now…a Magnetic Resonance Green Function?

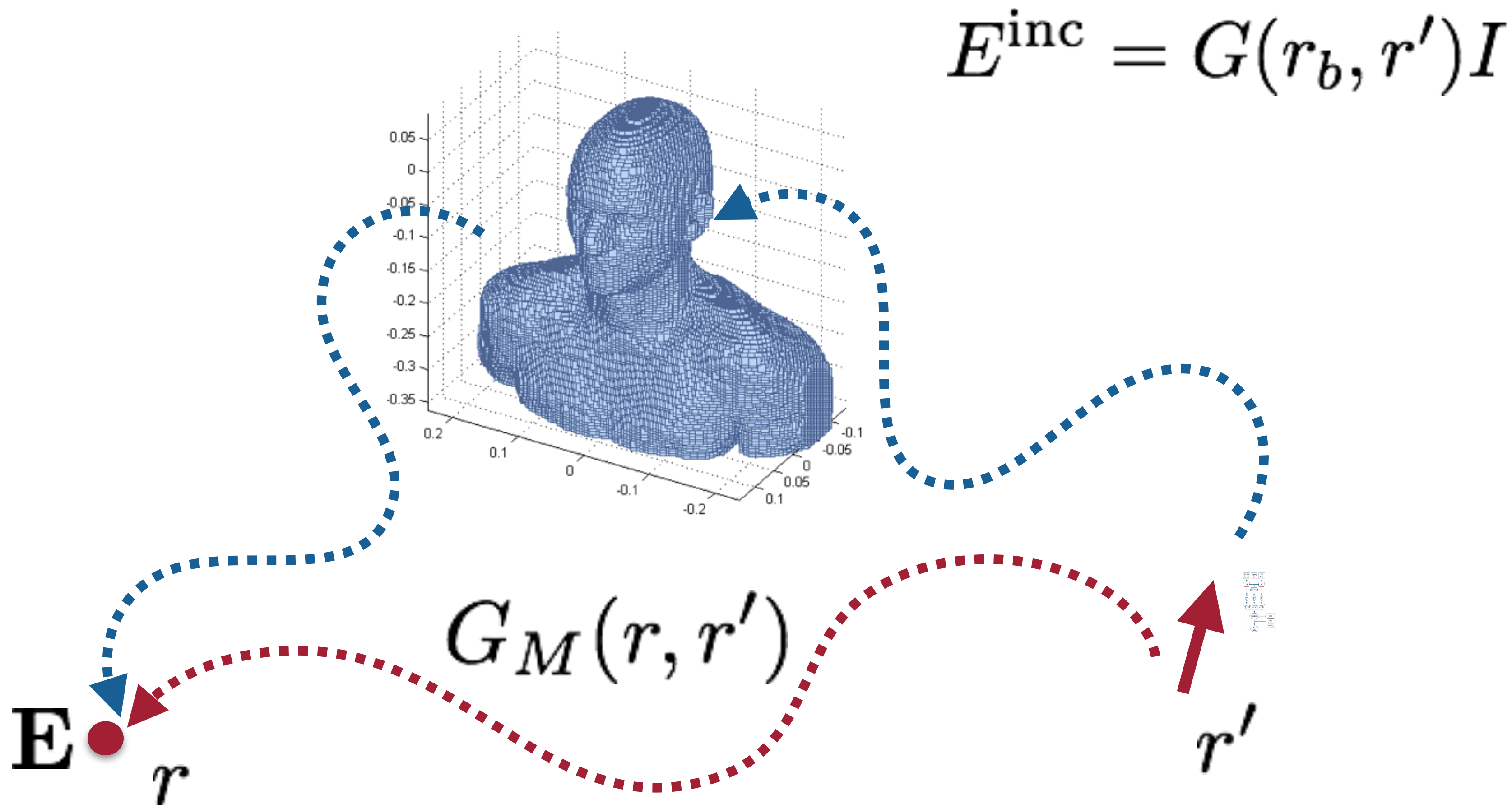
- **Magnetic Resonance Green function**
  - fundamental solution of body scattering problem at any point



- **Realistic human body model: too complex**
  - **Numerical Pre-computation!**
  - **Solution at any point due to any excitation**

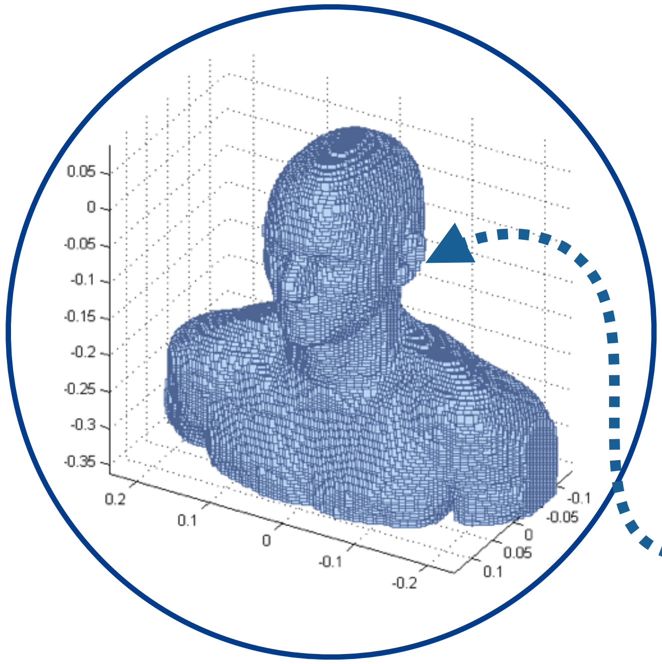
Now…a Magnetic Resonance Green Function?

- **How to explore all possibilities?**



Hyugens equivalent principle

- **Surface enclosing the scatterer**

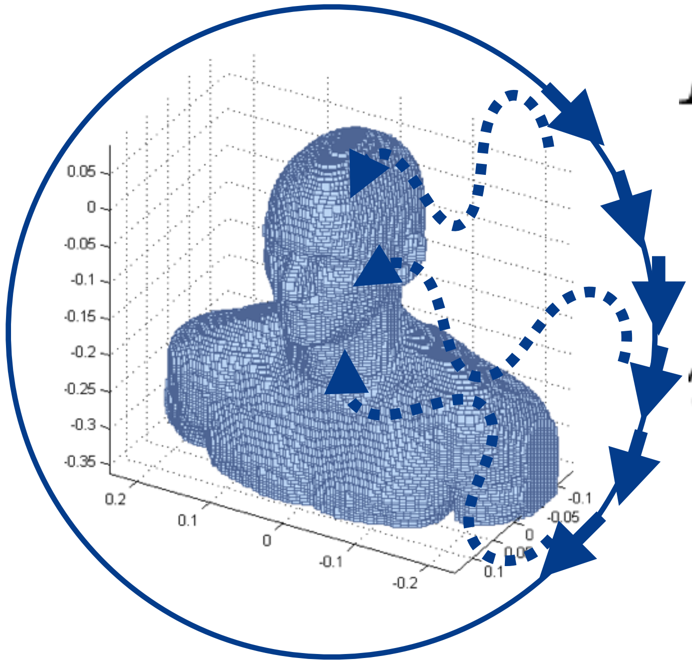


$$E^{\mathrm{inc}} = G(r_b, r')I$$

r'

Hyugens equivalent principle

- **Fields from (combination of effect of) currents on the surface**



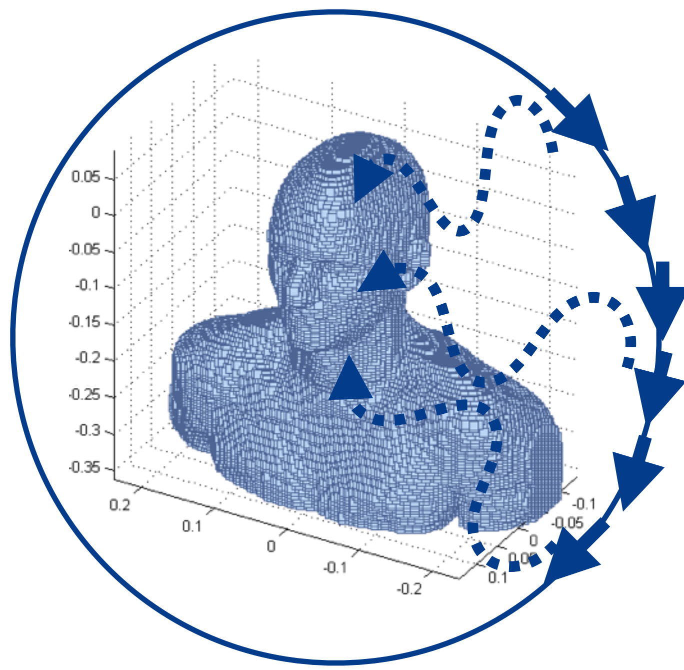
$$E^{\text{inc}} = \sum_{k}^{N_s} G(r_b, r_k) i_k \alpha_k$$

i_k

r'

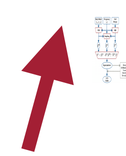
Hyugens equivalent principle

- **Fields from (incident fields due to) currents on the surface**



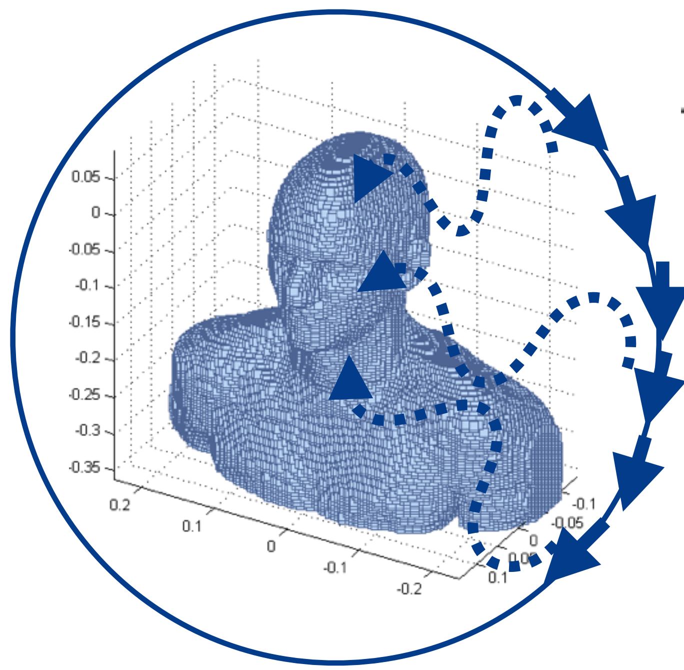
$$E^{\text{inc}} = \sum_{k}^{N_s} E_k^{\text{inc}} \alpha_k$$

$$i_k \longleftrightarrow E_k^{\text{inc}} = G(r_b, r_k) i_k$$



r'

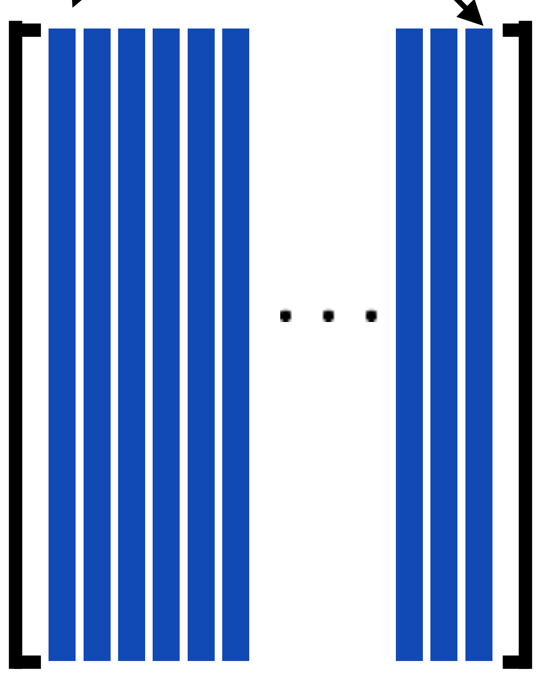
- **Find a basis for the incident fields**



$$E^{\mathrm{inc}} = \sum_{k}^{q} U_k \alpha_k$$

$$i_k \longleftrightarrow E_k^{\mathrm{inc}} = G(r_b, r_k) i_k$$

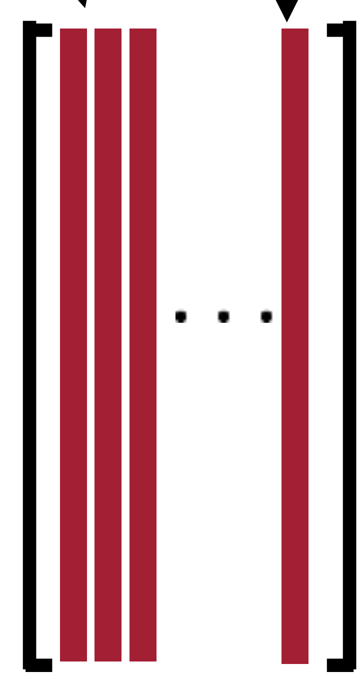
$E_1^{\mathrm{inc}} \dots E_{N_s}^{\mathrm{inc}}$



SVD

$U =$

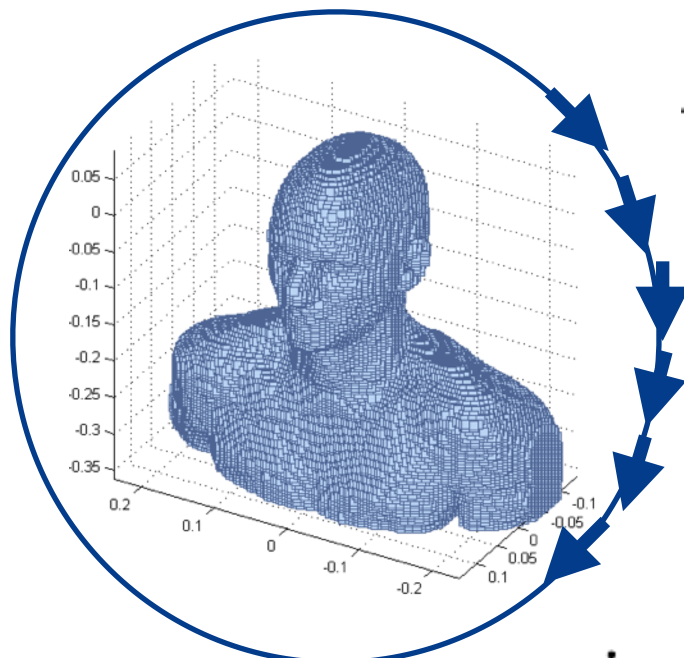
$U_1 \dots U_q$



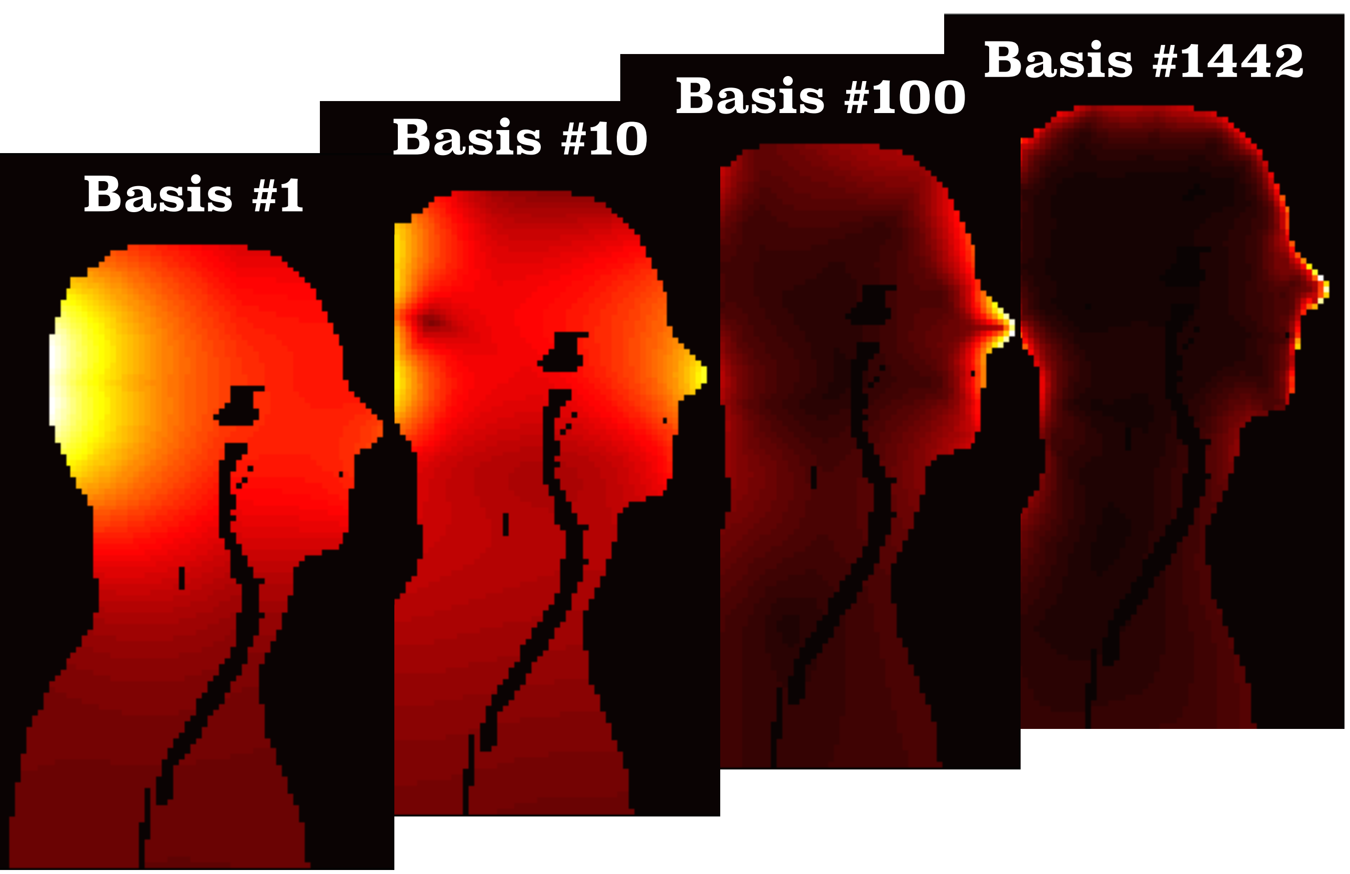
$$q << N_s$$
$$q << N_b$$

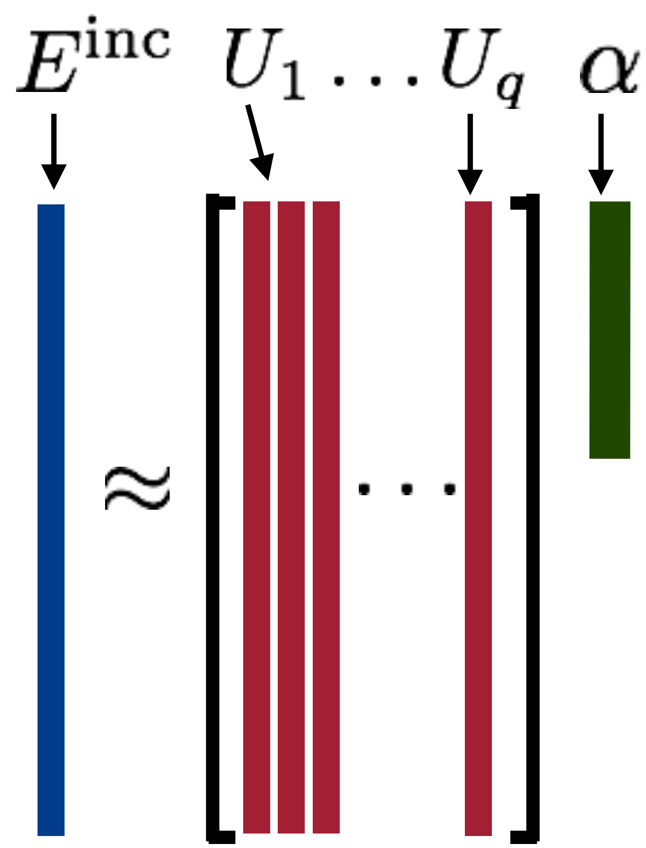
Basis for the incident field

- **Incident field approximated by basis**



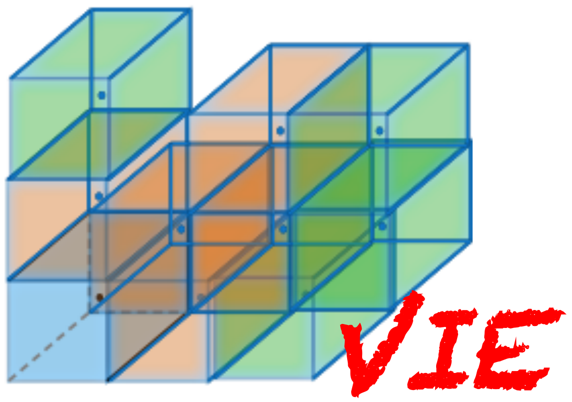
$$E^{\mathrm{inc}} = \sum_{k}^{q} U_k \alpha_k$$

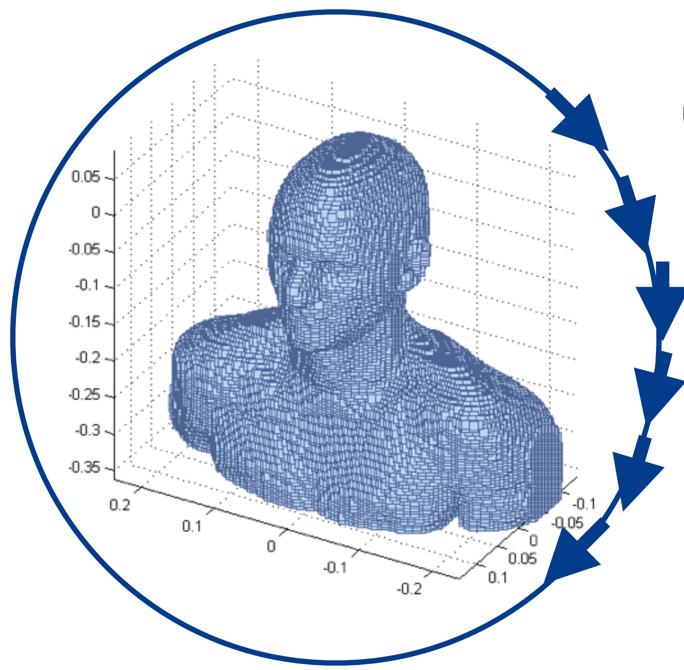




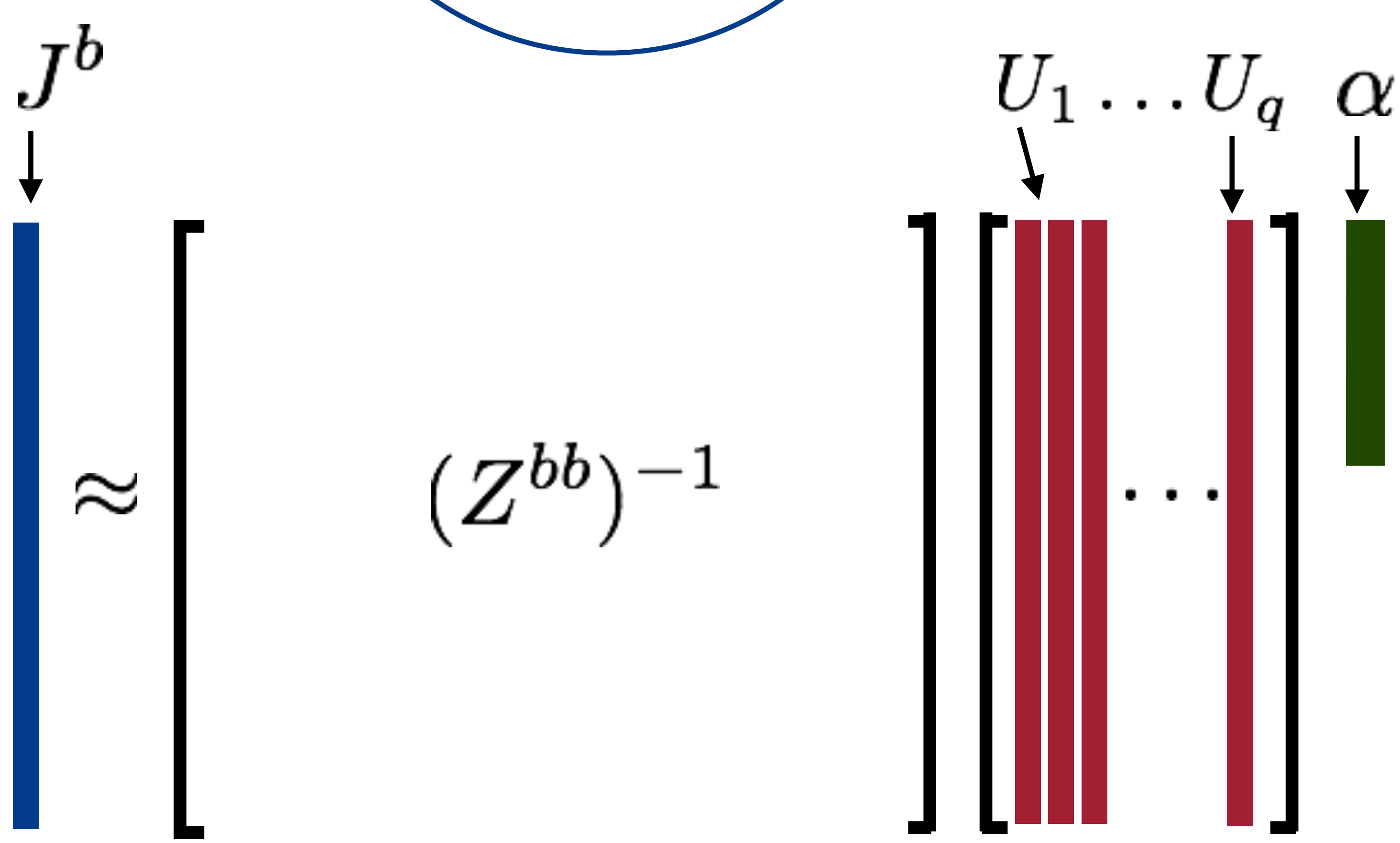
Basis for the incident field

- **Pre-compute the VIE solution**



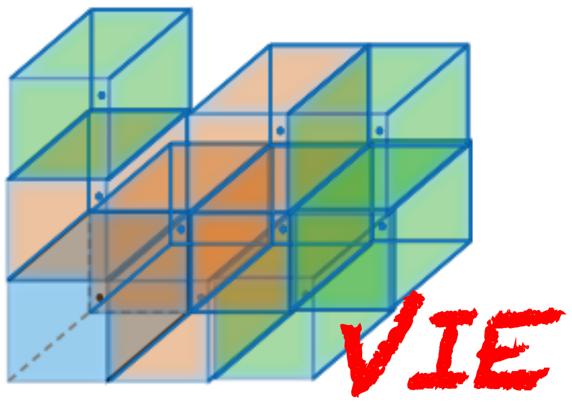


$$J^b = (Z^{bb})^{-1} E^{\text{inc}}$$

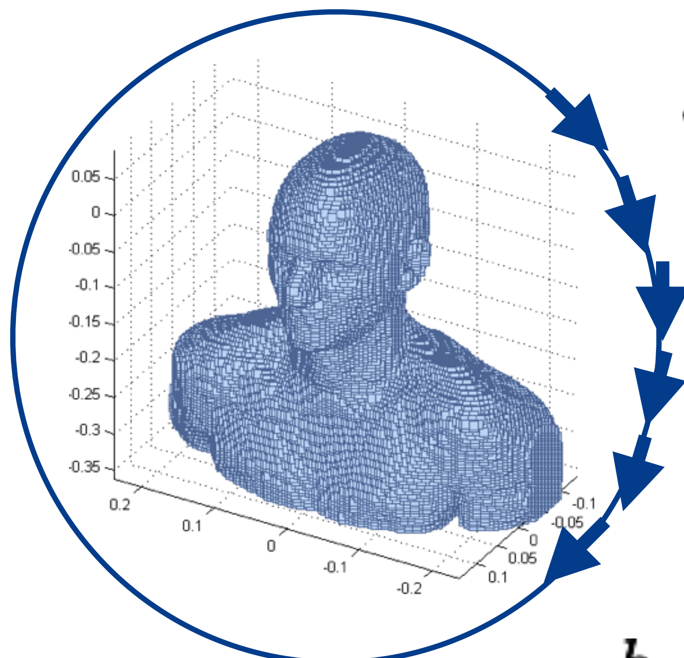


Basis for the solution

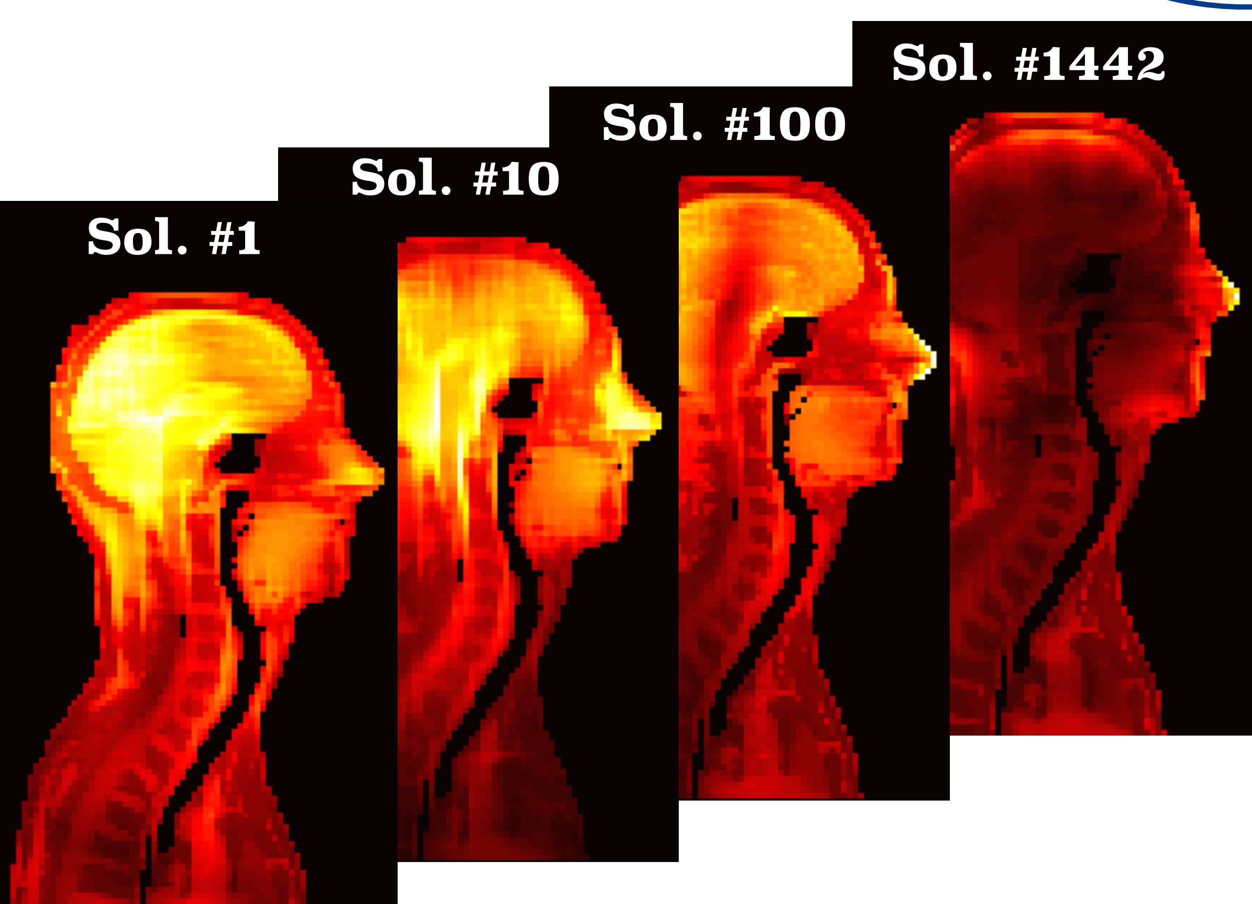
- **Pre-compute the VIE solution for each vector in basis**

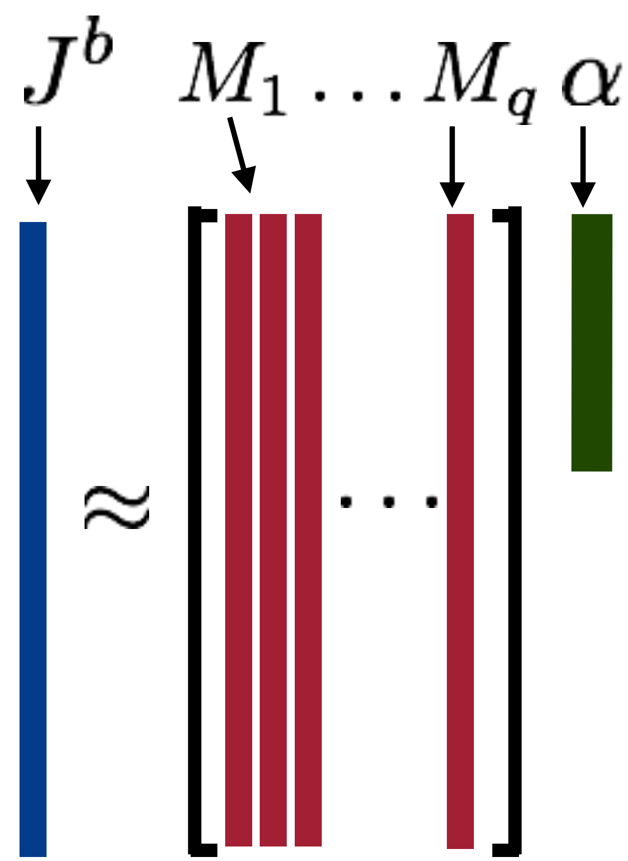


$$M_k = (Z^{bb})^{-1} U_k$$



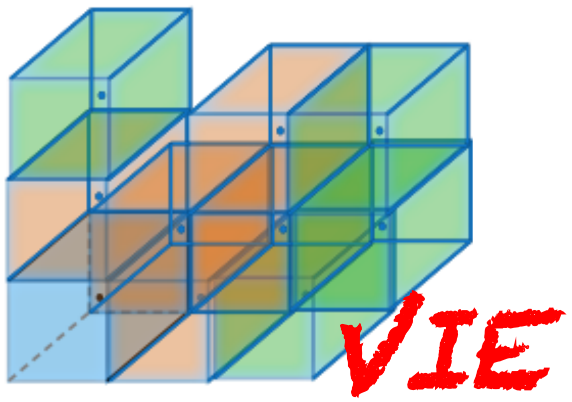
$$J^b \approx \sum_k^q (Z^{bb})^{-1} U_k \alpha_k$$

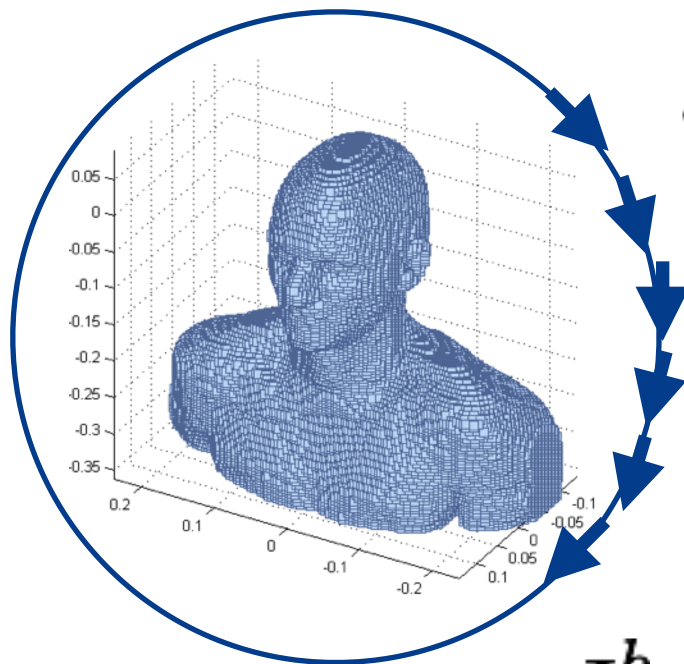




Basis for the solution

- **Pre-compute the VIE solution for each vector in basis**





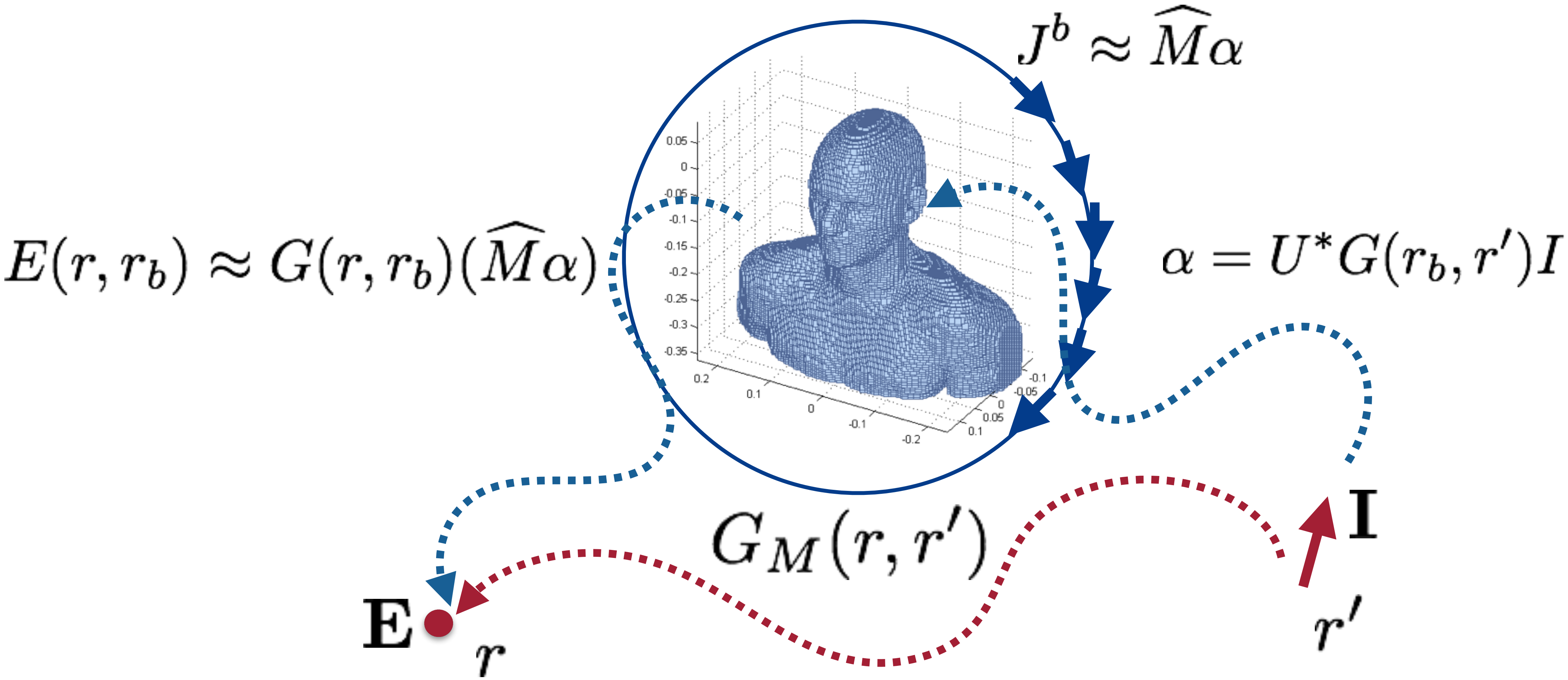
$$J^b \approx \sum_k^q (Z^{bb})^{-1} U_k \alpha_k$$

$$M_k = (Z^{bb})^{-1} U_k$$

$$J^b \approx \left[\widehat{M} \right] \alpha$$

Magnetic resonance Green functions

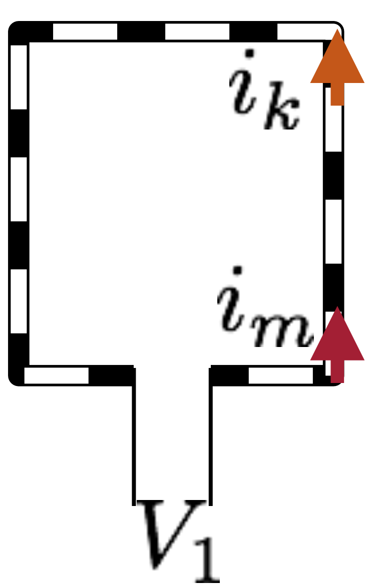
- **Contribution of human body as set of matrix-vector products**

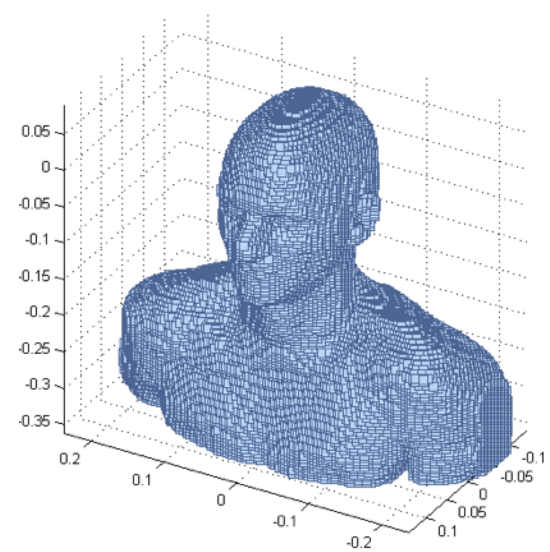


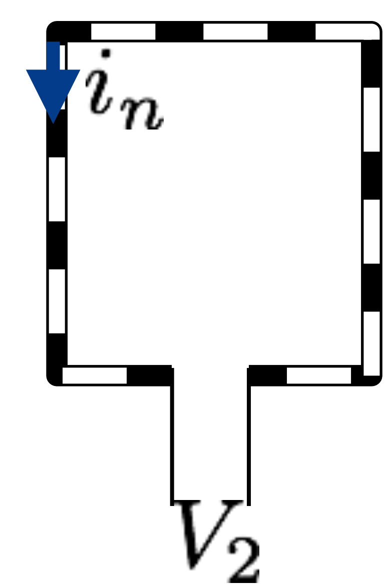
How to use the basis

Accelerated integral equation solver

- **Accelerate the integral equation solver**



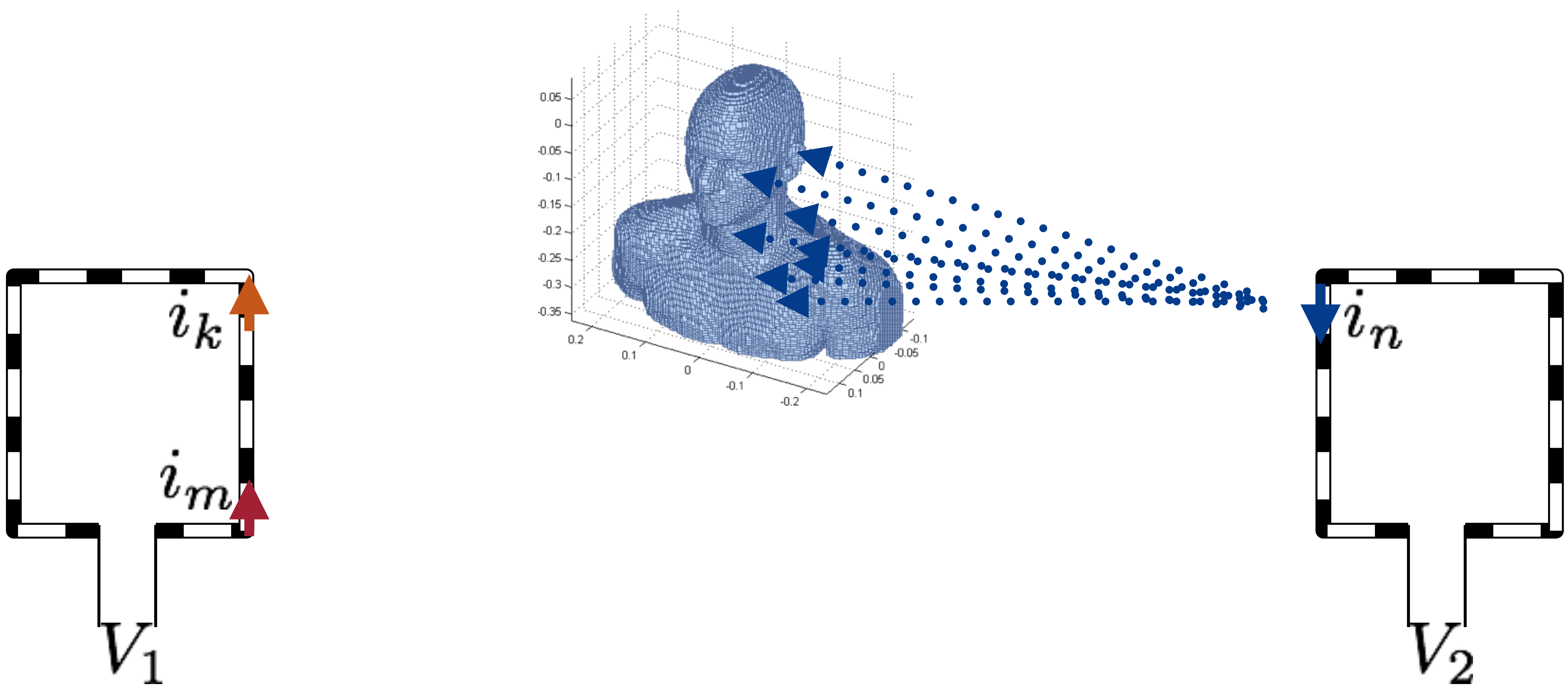




Accelerated IE solver by MRGFs

- **Model the body perturbation with MRGFs**

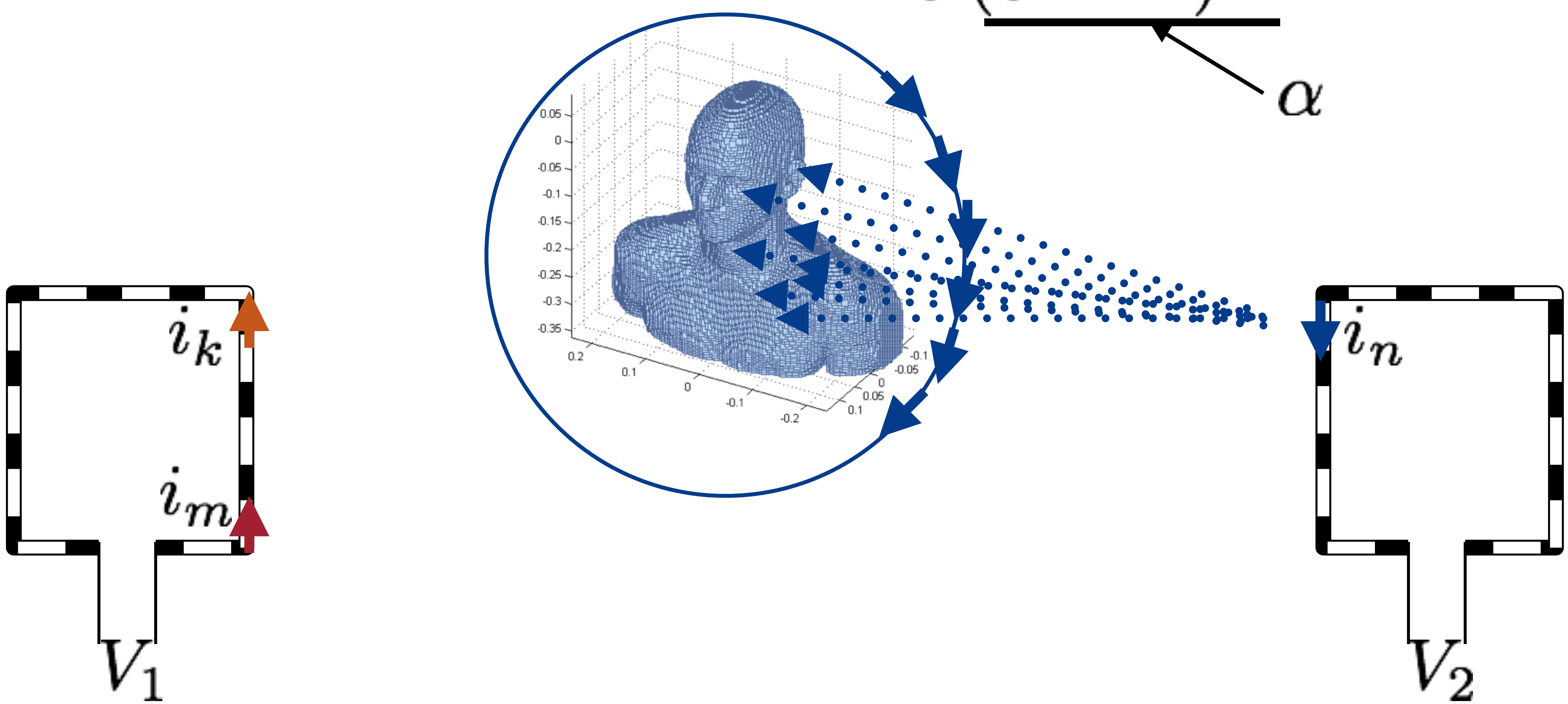
$$E^{\text{inc}} = Z^{bc} I^{c}$$



- **Model the body perturbation with MRGFs**

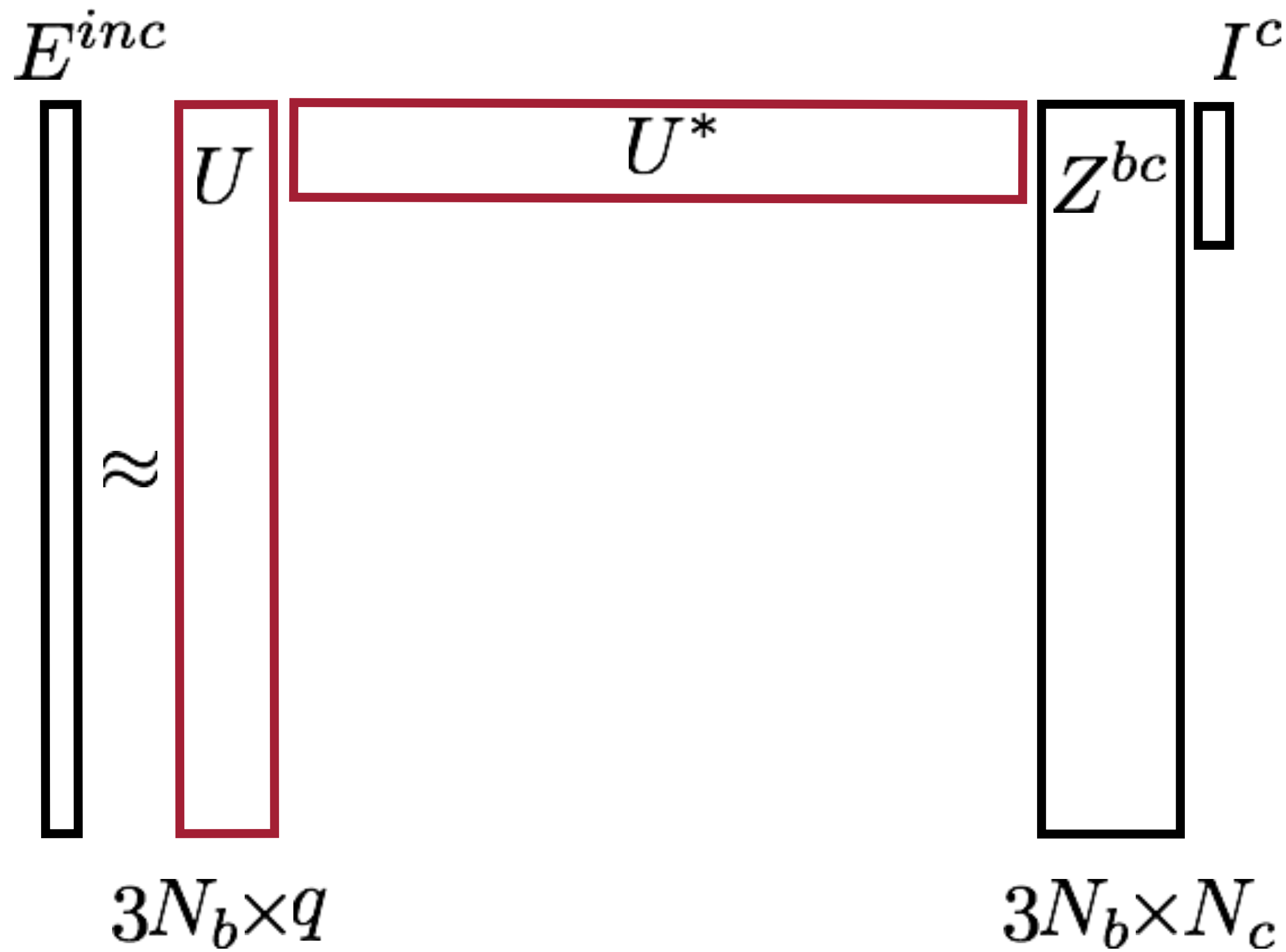
$$E^{\mathrm{inc}} \approx U\underline{(U^{*}Z^{bc})}I^{c}$$

α



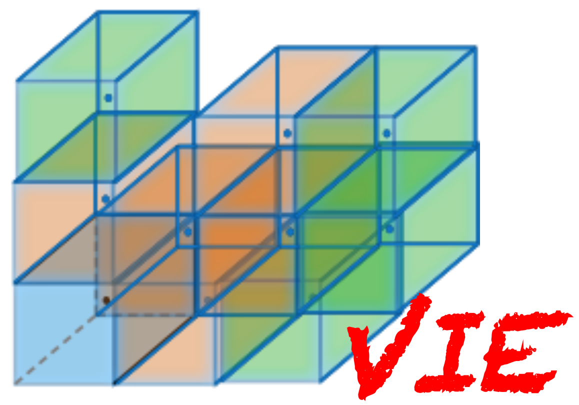
Accelerated IE solver by MRGFs

- **Model the body perturbation with MRGFs**
  - approximate incident field by basis

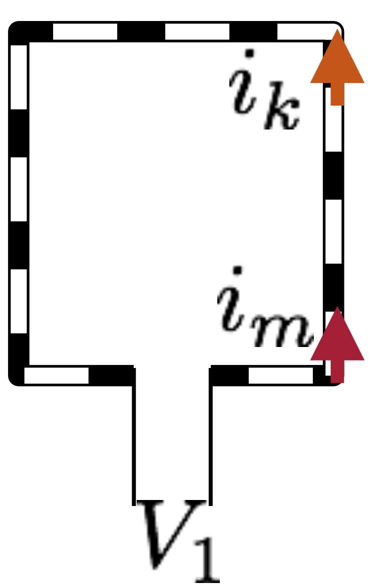


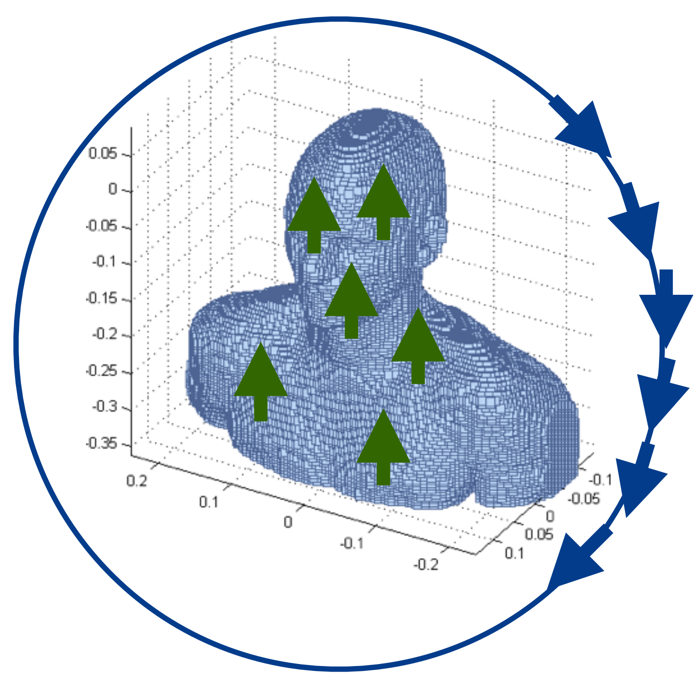
Accelerated IE solver by MRGFs

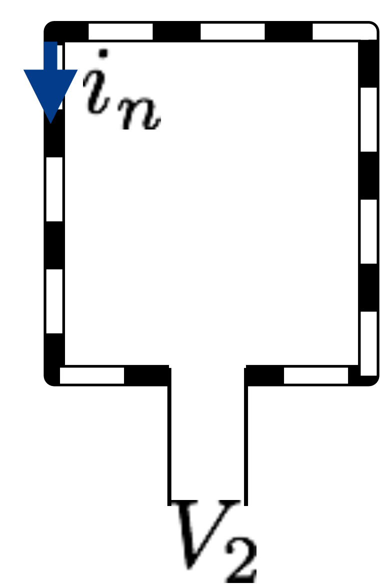
- **Model the body perturbation with MRGFs**



$$J^b = (Z^{bb})^{-1}E^{\text{inc}} \approx \underbrace{(Z^{bb})^{-1}U}_{\widehat{M}}\underbrace{(U^*Z^{bc})I^c}_{\alpha}$$

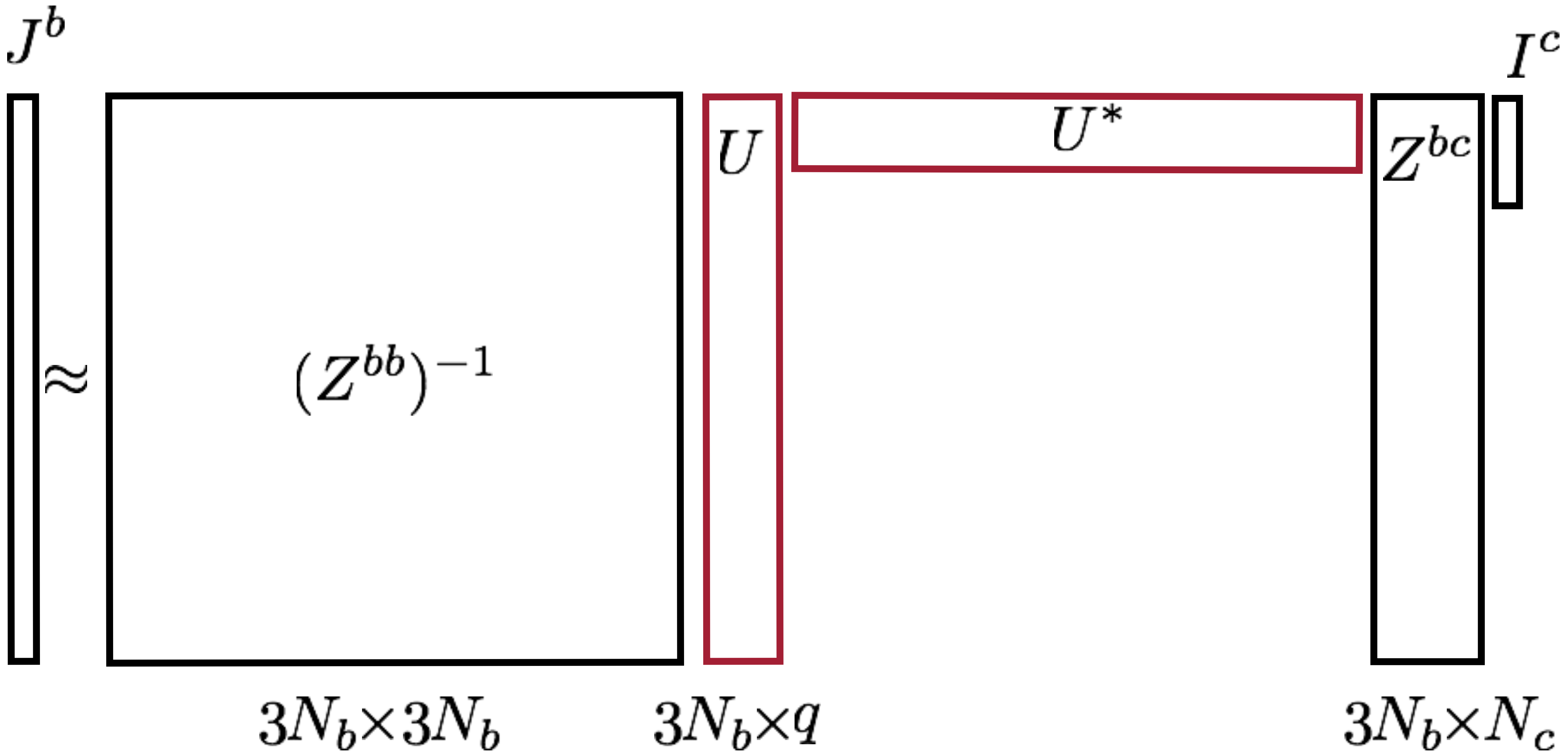




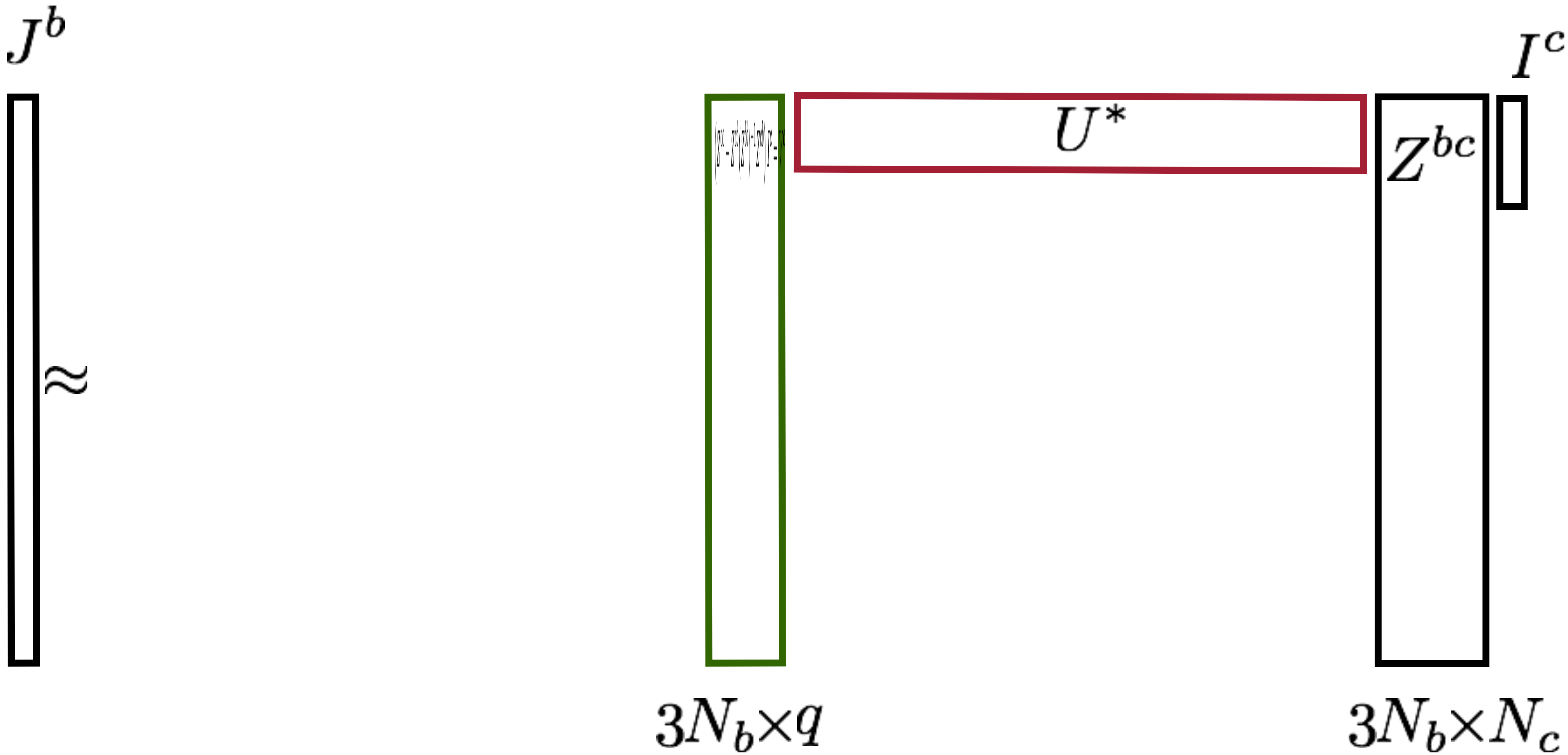


Accelerated IE solver by MRGFs

- **Model the body perturbation with MRGFs**
  - instead of applying the VIE solver



- **Model the body perturbation with MRGFs**
  - apply the pre-computed solutions



Accelerated IE solver by MRGFs

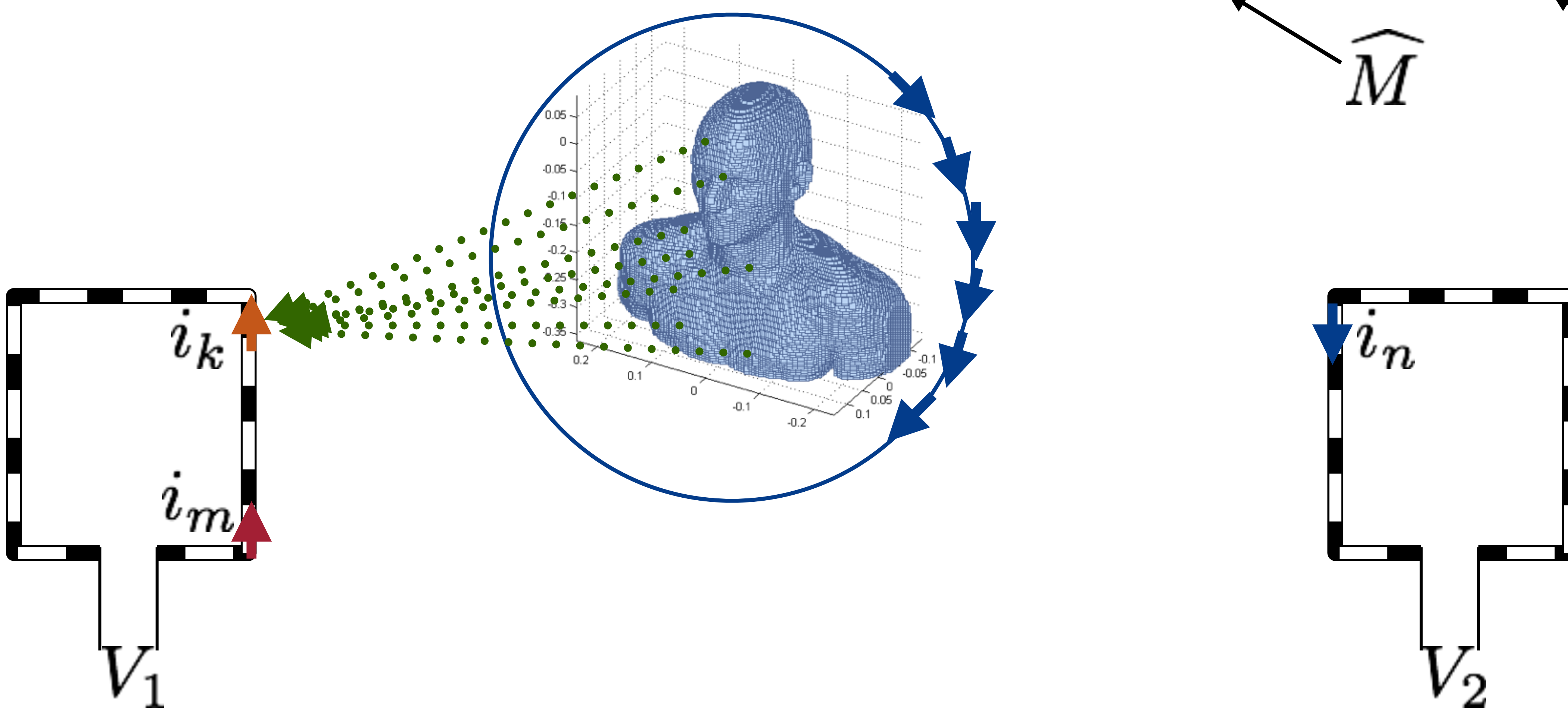
- **Model the body perturbation with MRGFs**

q x q precomputed matrix

$$\widehat{Z^{bb}}^{-1}$$

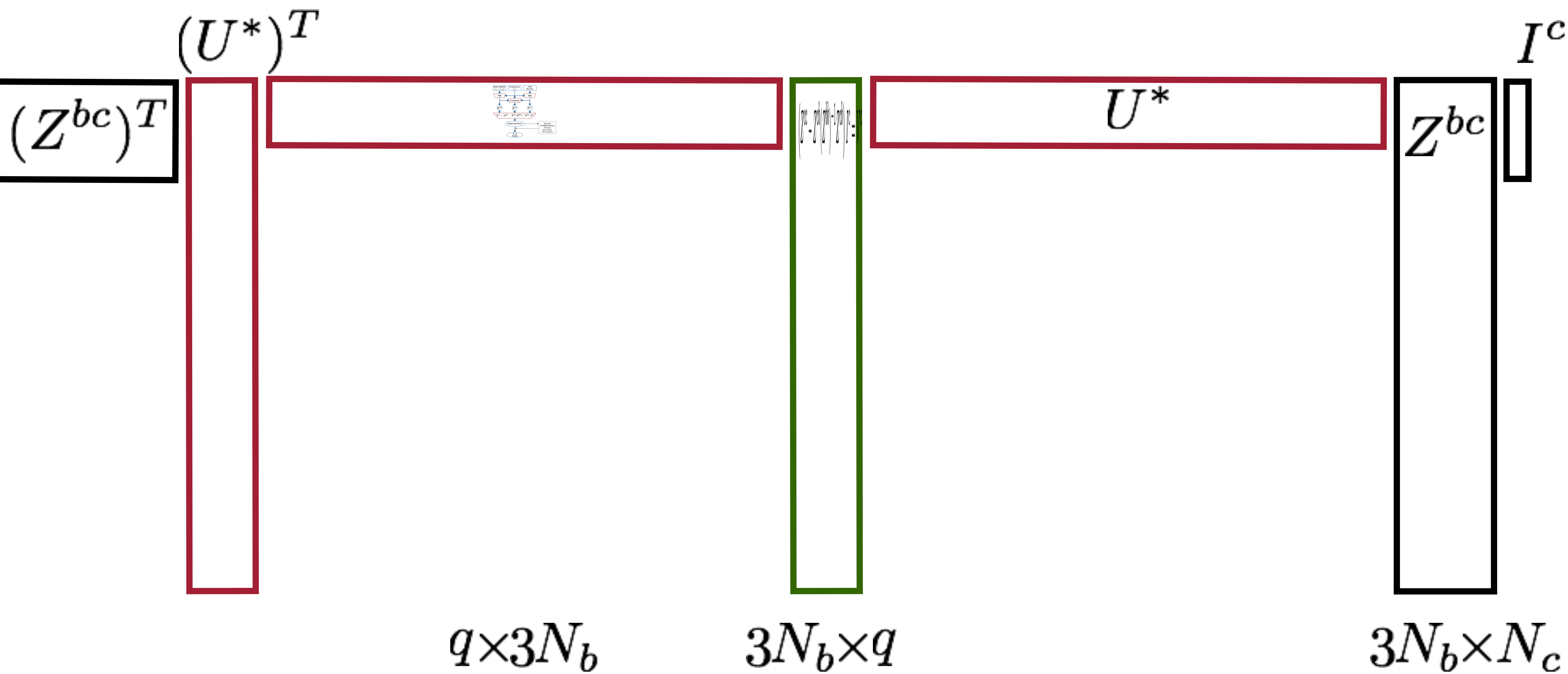
$$Z^p \approx (U^* Z^{bc})^T \overline{U^T (Z^{bb})^{-1} U} (U^* Z^{bc}) I^c$$

$\widehat{M}$ α



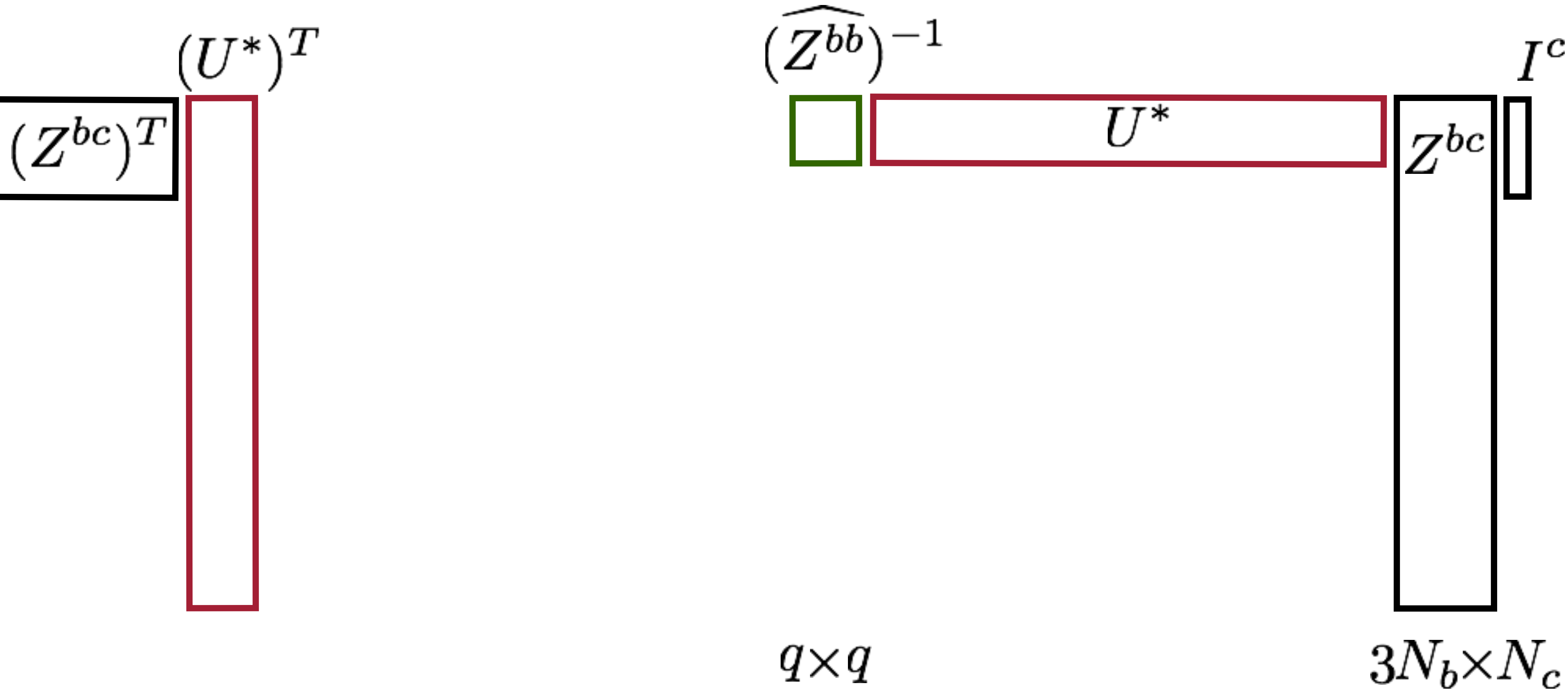
Accelerated IE solver by MRGFs

- **Model the body perturbation with MRGFs**
  - exploit reciprocity of Green functions on left side



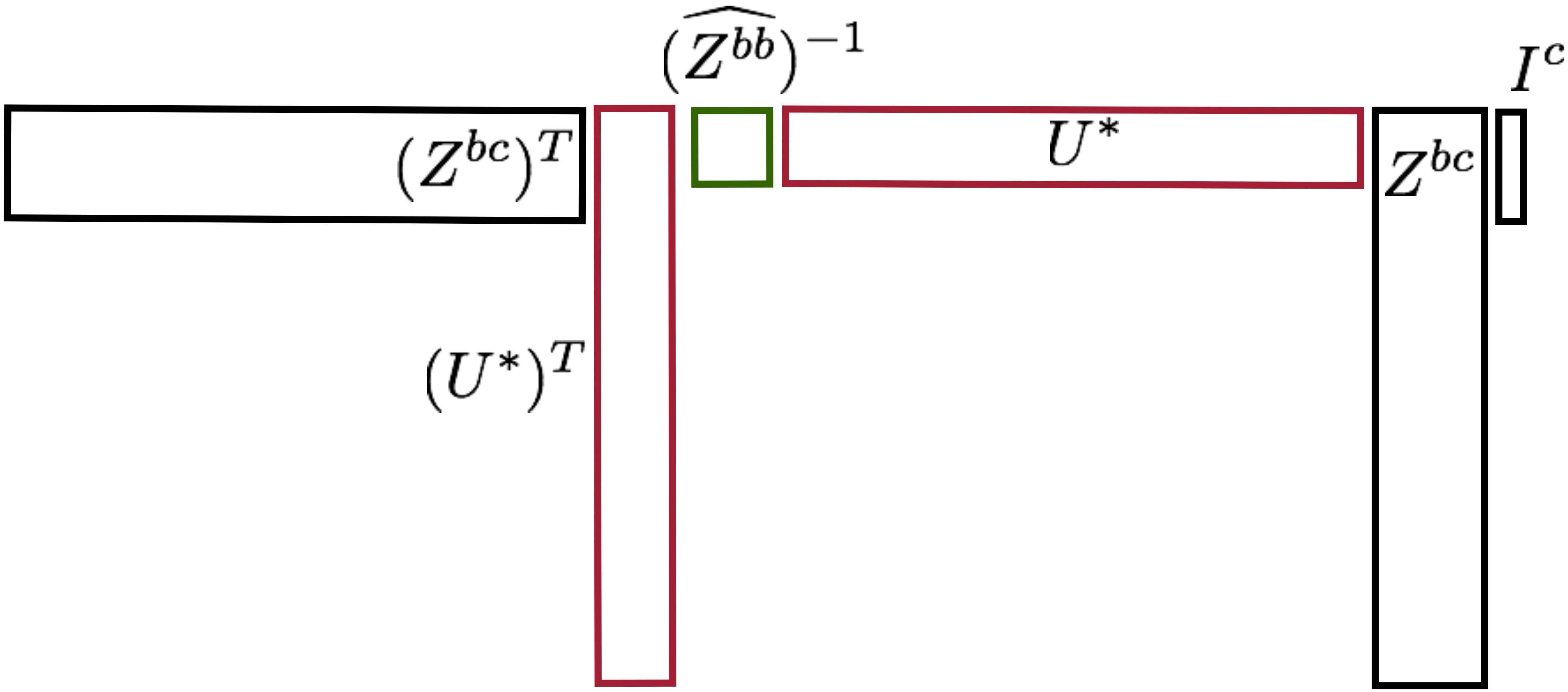
Accelerated IE solver by MRGFs

- **Model the body perturbation with MRGFs**
  - apply further compression



Accelerated IE solver by MRGFs

- **Model the body perturbation with MRGFs**
  - to generate the perturbation matrix $N_c \times N_c$

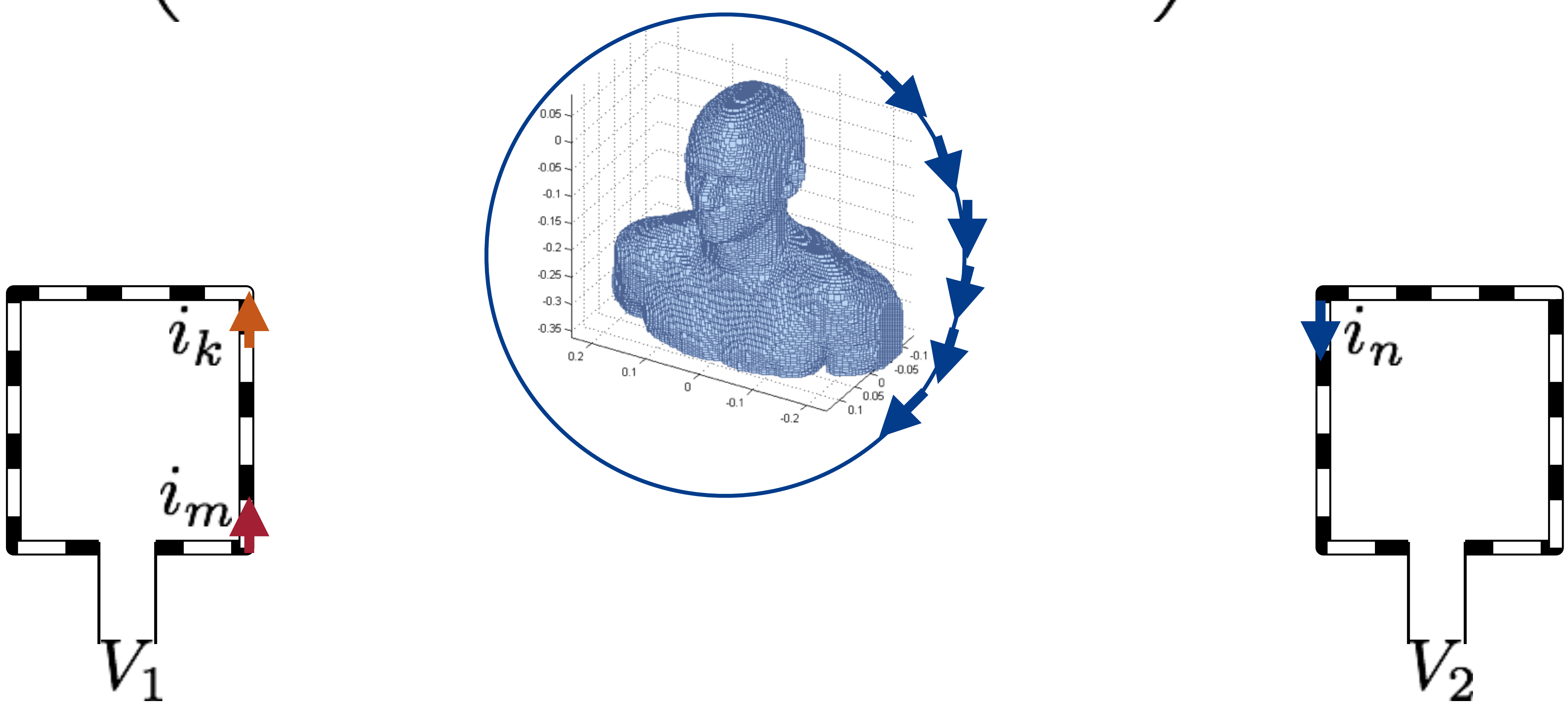


Accelerated IE solver by MRGFs

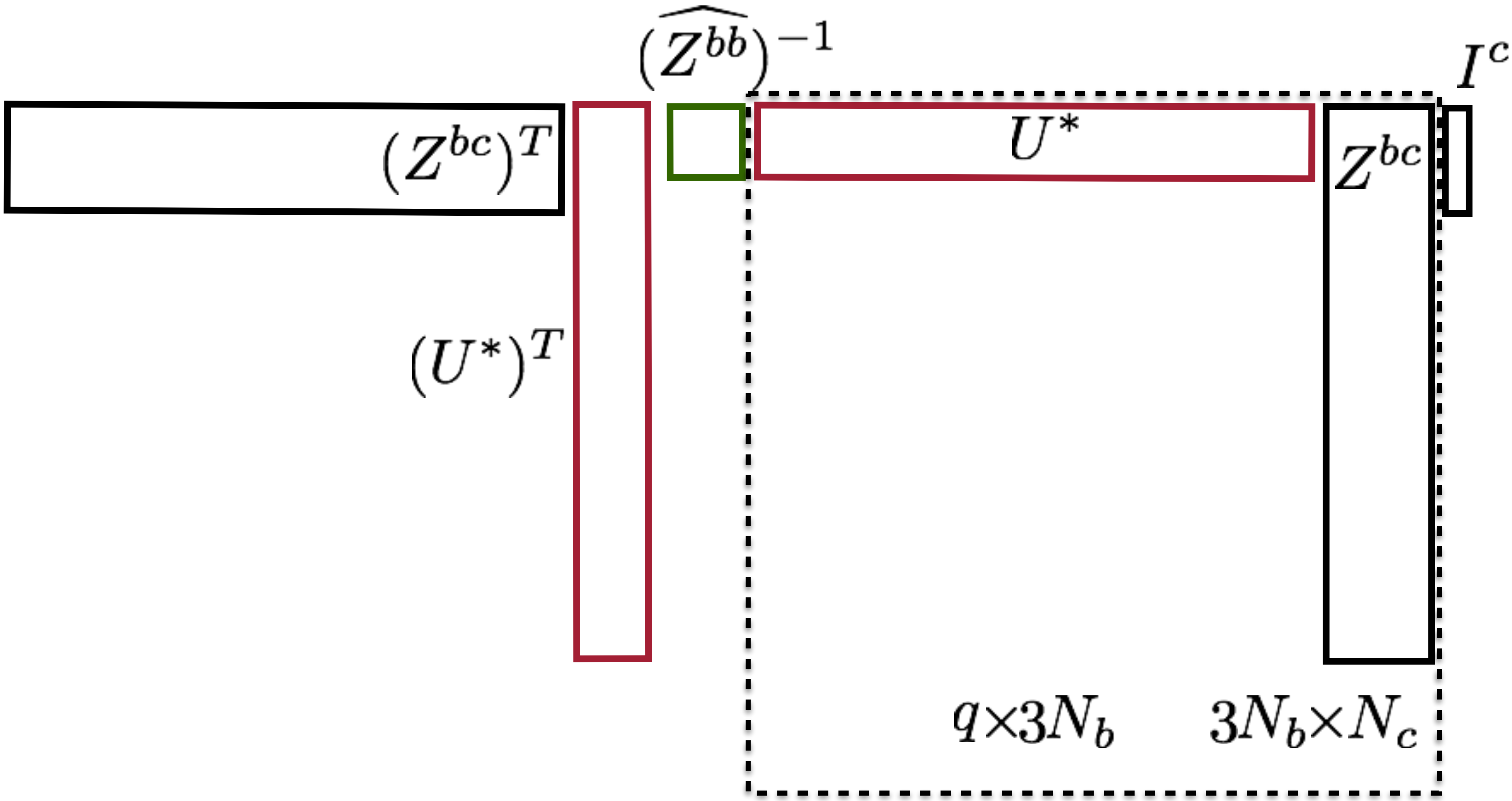
- **Model the body perturbation with MRGFs**
  - avoids VIE solver
  - assemble the perturbation: a set of low-rank matrix-vector products

q x q precomputed matrix

$$\left(Z^{cc} - (U^{*} Z^{bc})^{T} \widehat{Z^{bb}}^{-1} (U^{*} Z^{bc}) \right) I^{c} = V^{c}$$



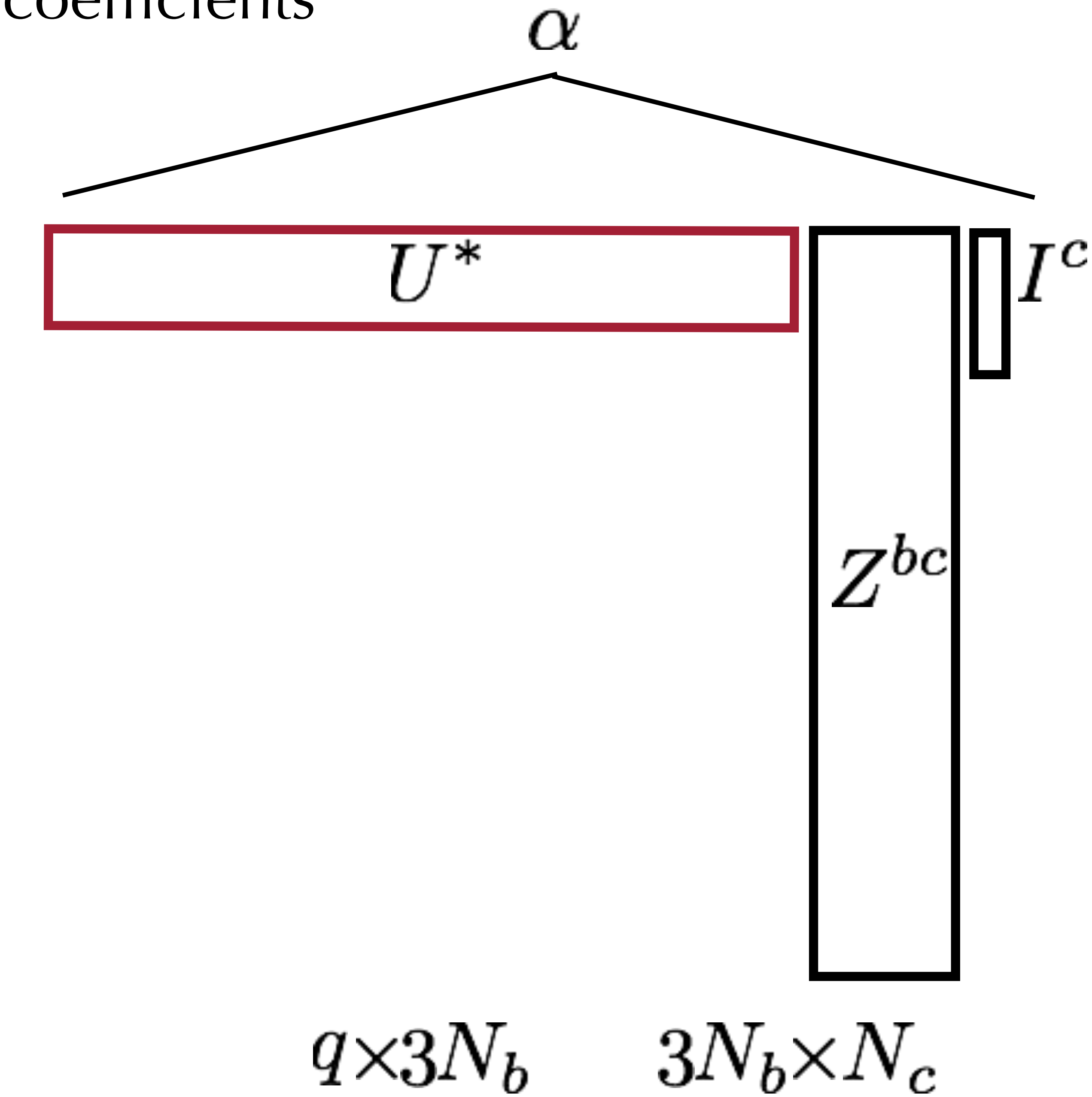
- **Still requires to form and project the coupling block**
  - for every new coil



Accelerated IE solver by MRGFs

- **Still requires to form and project the coupling block**
  - approximate the computation of the coefficients

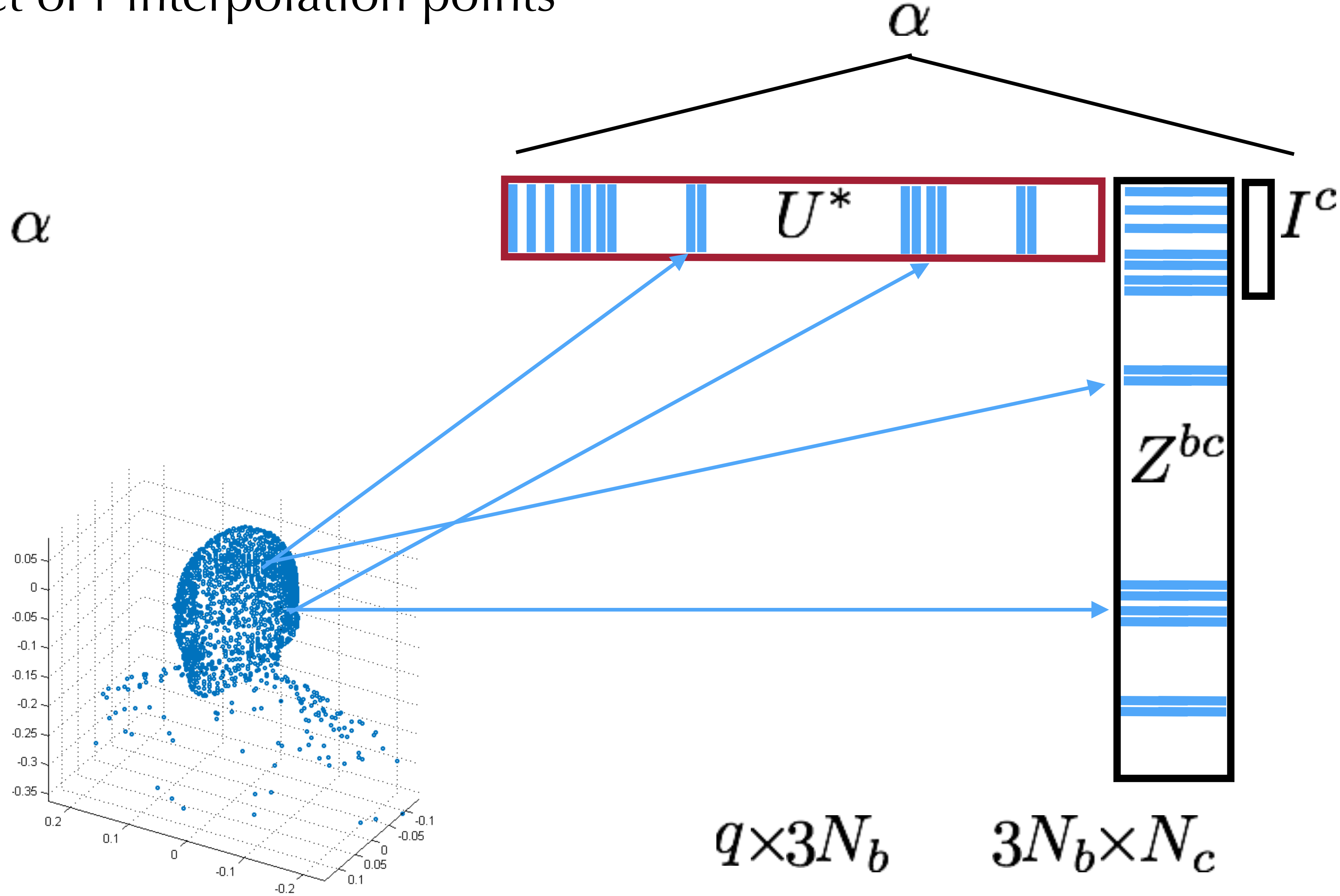
$$U^* Z^{bc} I^c = \alpha$$



Accelerated IE solver by MRGFs

- **Still requires to form and project the coupling block**
  - using a small set of r interpolation points

$$U^* Z^{bc} I^c = \alpha$$

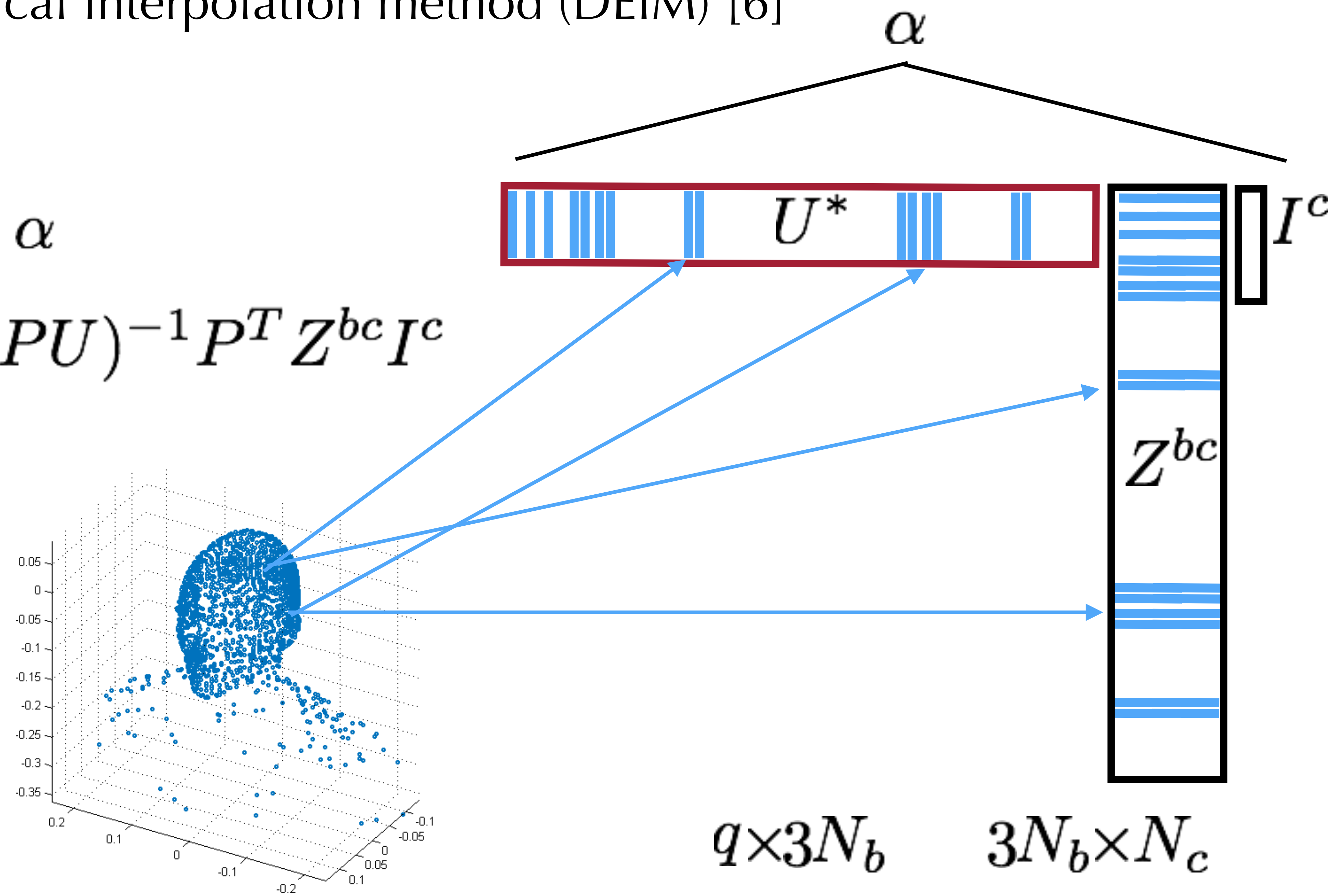


Accelerated IE solver by MRGFs

- **Still requires to form and project the coupling block**
  - Discrete empirical interpolation method (DEIM) [6]

$$U^* Z^{bc} I^c = \alpha$$

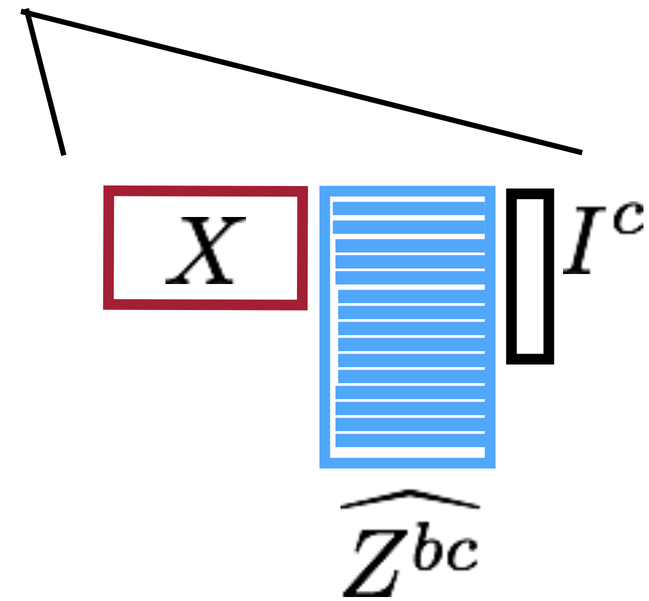
$$\alpha \approx \alpha_p = (PU)^{-1} P^T Z^{bc} I^c$$



[6] **Chaturantabut** et al, “Nonlinear model reduction via discrete empirical interpolation,” ***SIAM J. Sci. Comput.*** **2010.**

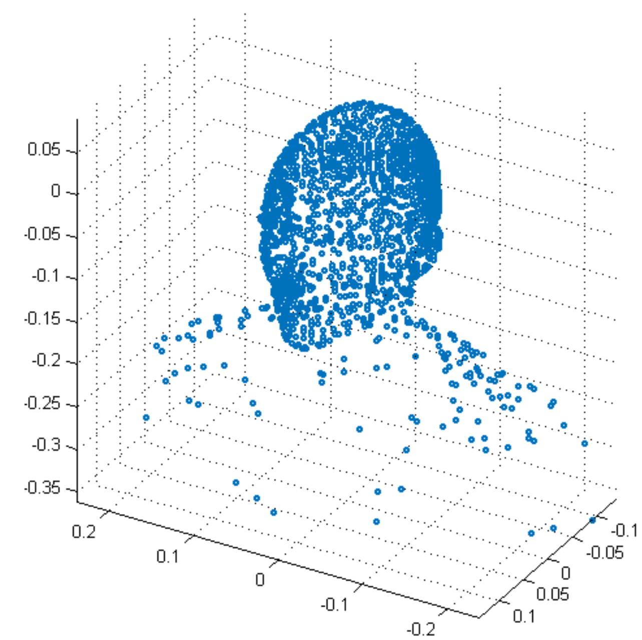
- **Still requires to form and project the coupling block**
  - pre-compute the matrix X
  - only need to evaluate coupling at r points

$$\alpha \approx \alpha_p = (PU)^{-1} P^T Z^{bc} I^c$$



$$X = (PU)^{-1}$$

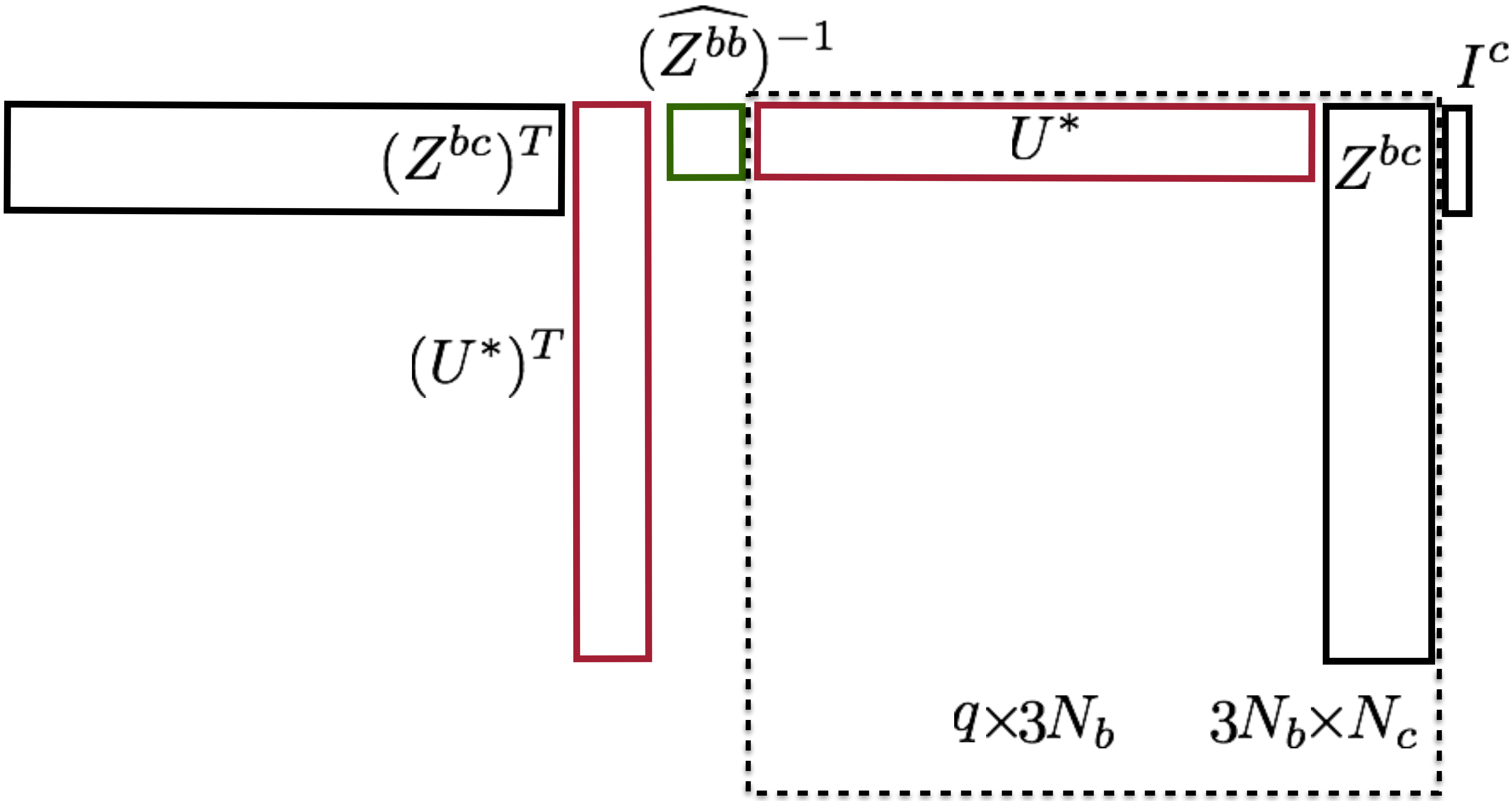
$$\widehat{Z^{bc}} = P^T Z^{bc}$$



$q \times r \quad r \times N_c$

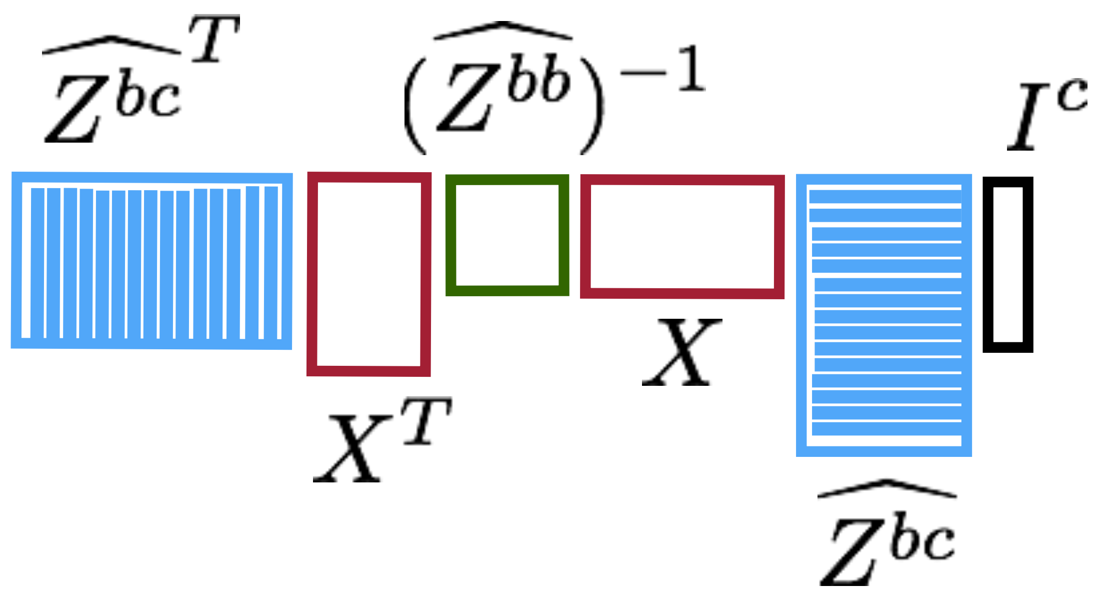
Accelerated IE solver by MRGFs

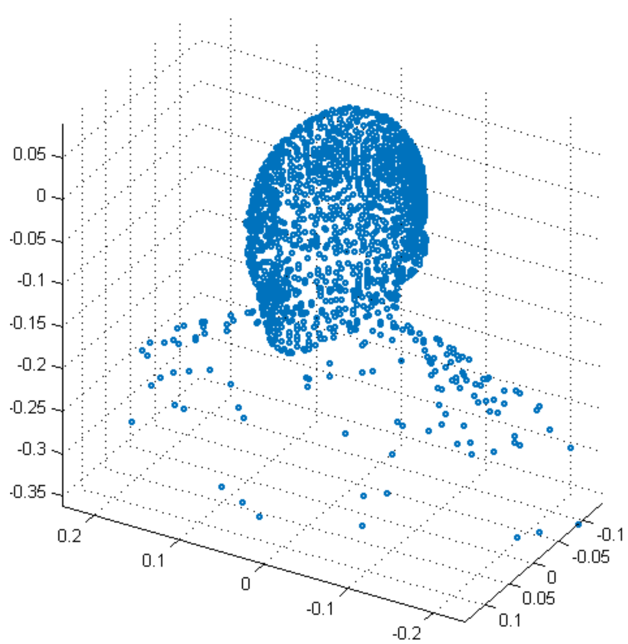
- **Instead of evaluating the coupling in all positions and project**



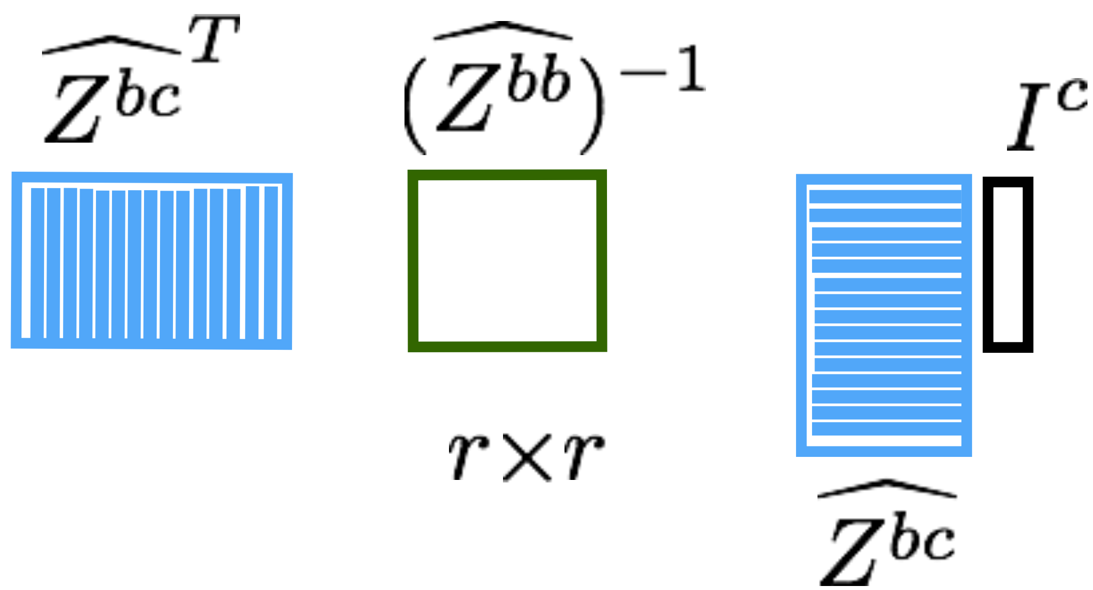
Accelerated IE solver by MRGFs

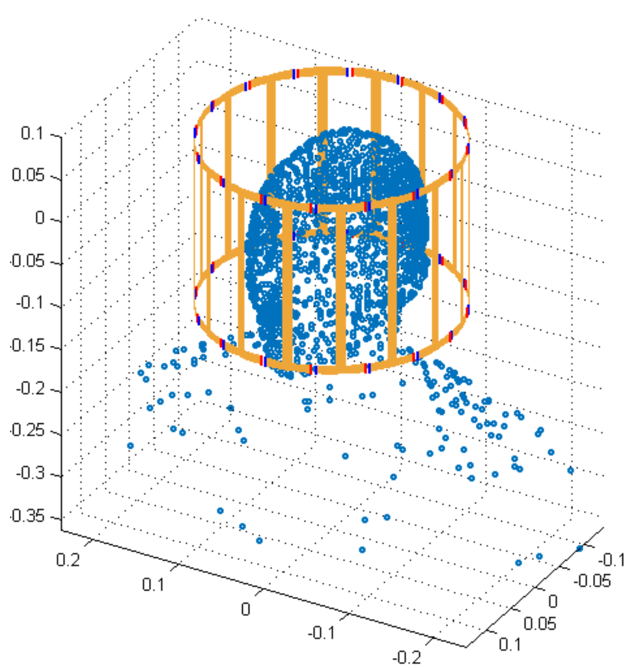
- **Instead of evaluating the coupling in all positions and project**
  - evaluation in r points





- **Instead of evaluating the coupling in all positions and project**
  - evaluation in r points

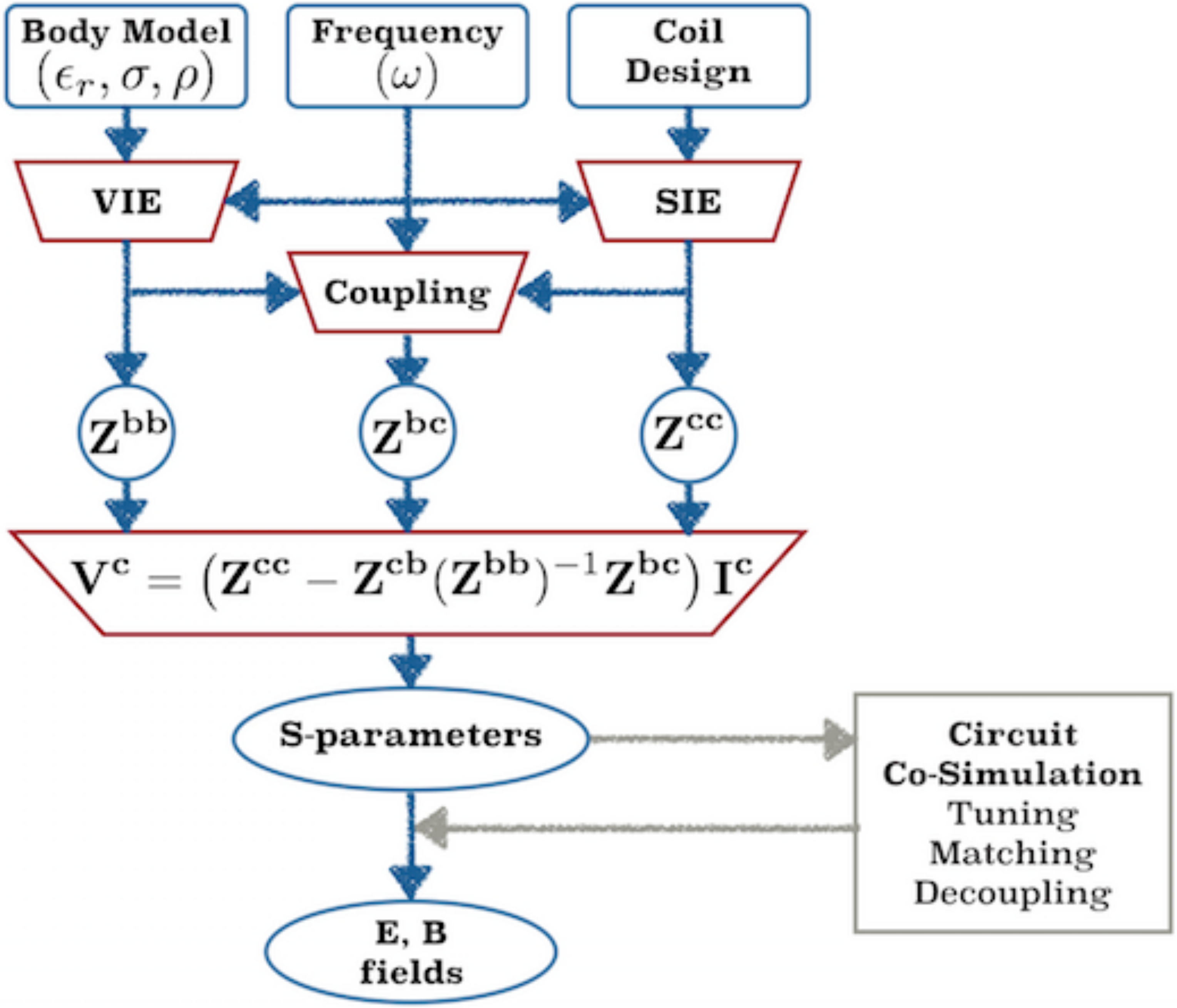




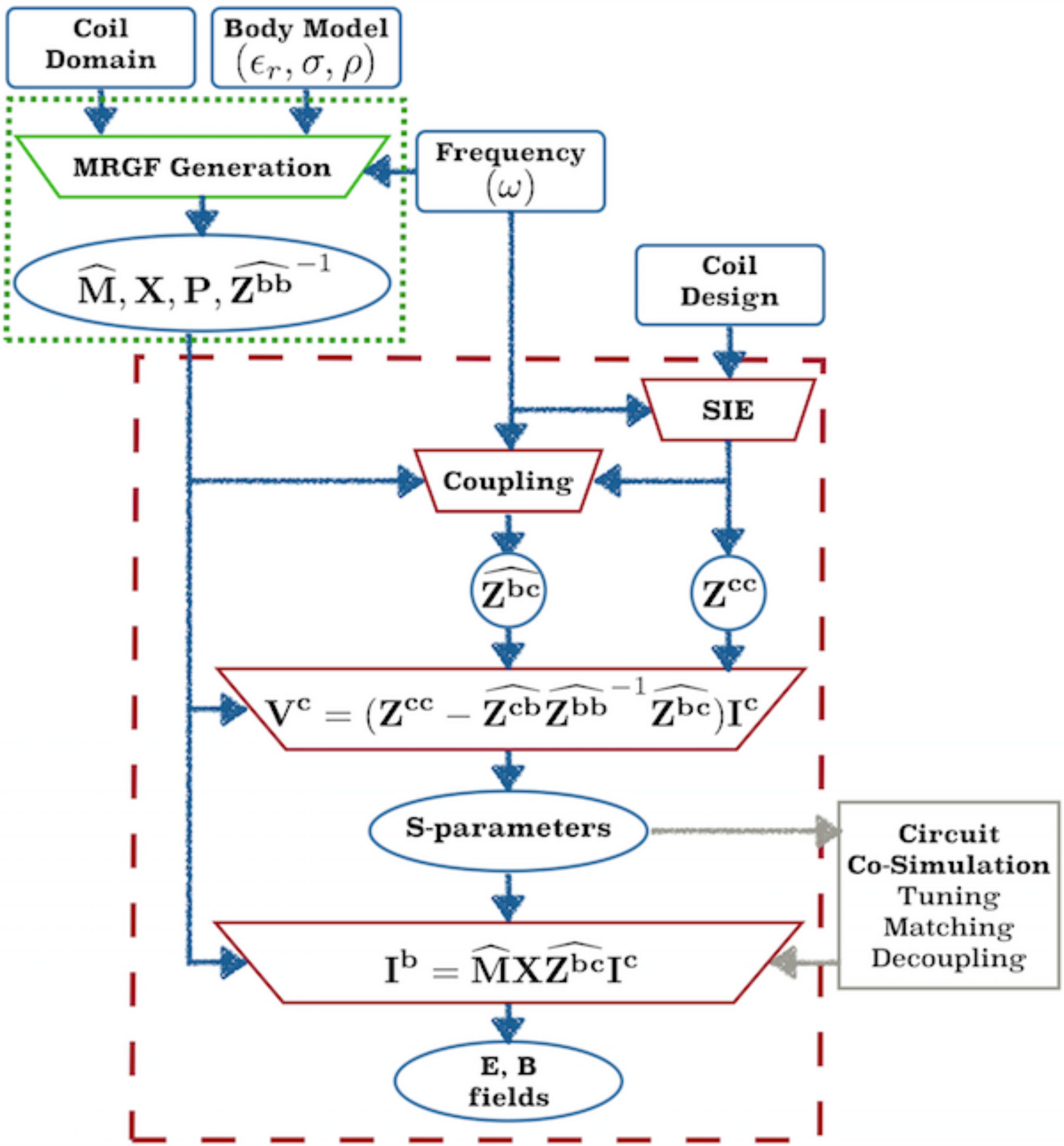
$$\left(Z^{cc} - \widehat{Z^{bc}}^T \widehat{Z^{bb}}^{-1} \widehat{Z^{bc}}\right) I^c = V^c$$

$$N_c \times N_c$$

IE solver flow



MRGFs accelerated IE solver flow



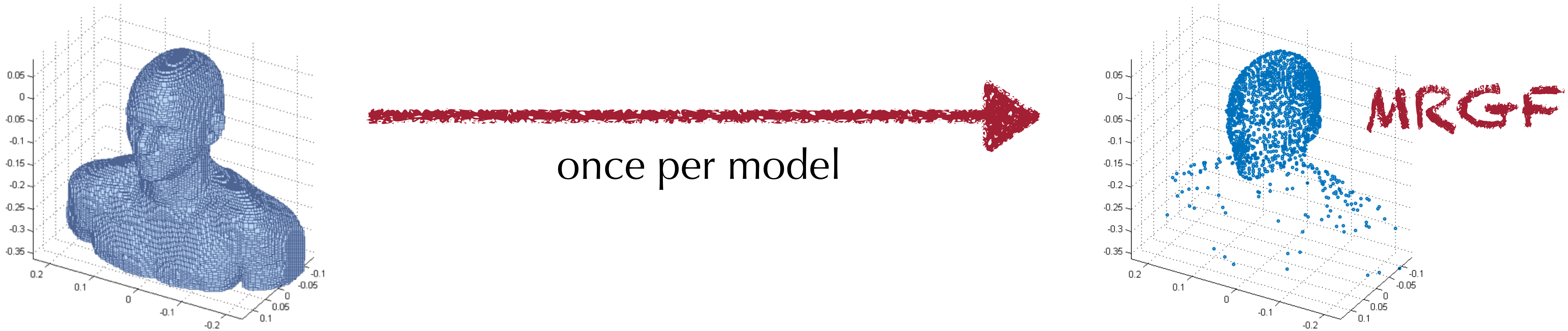
Two-Stage approach

- **Accelerated Integral Equation solver for MRI coils**
  - **Off-line phase**

  - **On-line phase**

Two-Stage approach

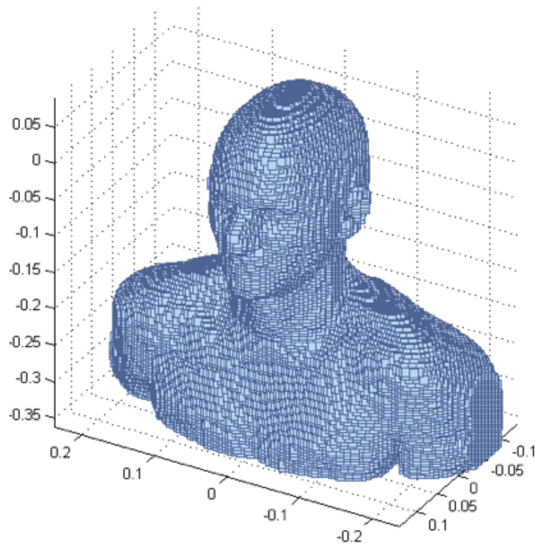
- **Accelerated Integral Equation solver for MRI coils**
  - **Off-line phase: MRGF pre-computation**



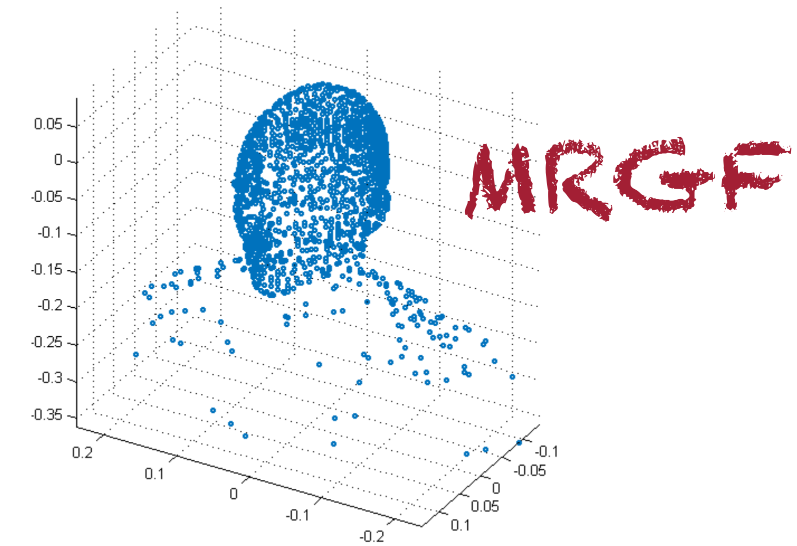
  - **On-line phase**

Two-Stage approach

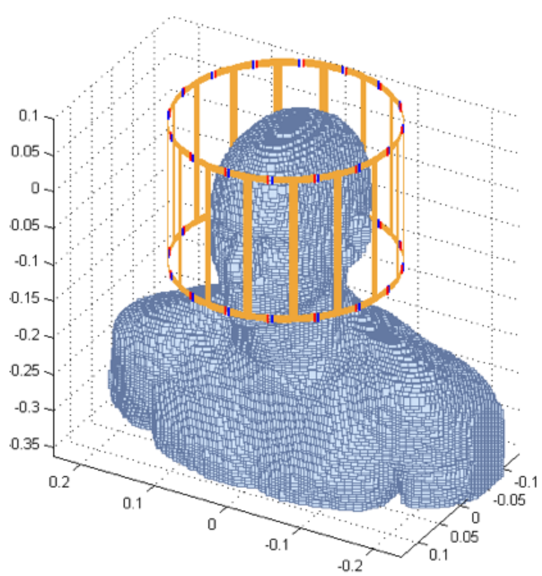
- **Accelerated Integral Equation solver for MRI coils**
  - **Off-line phase: MRGF pre-computation**



once per model



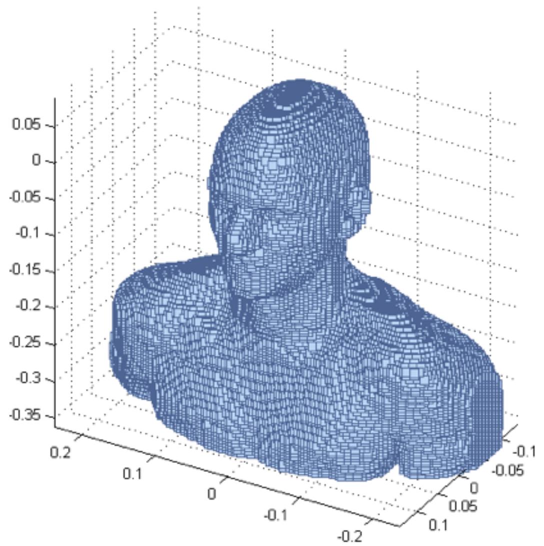
  - **On-line phase**



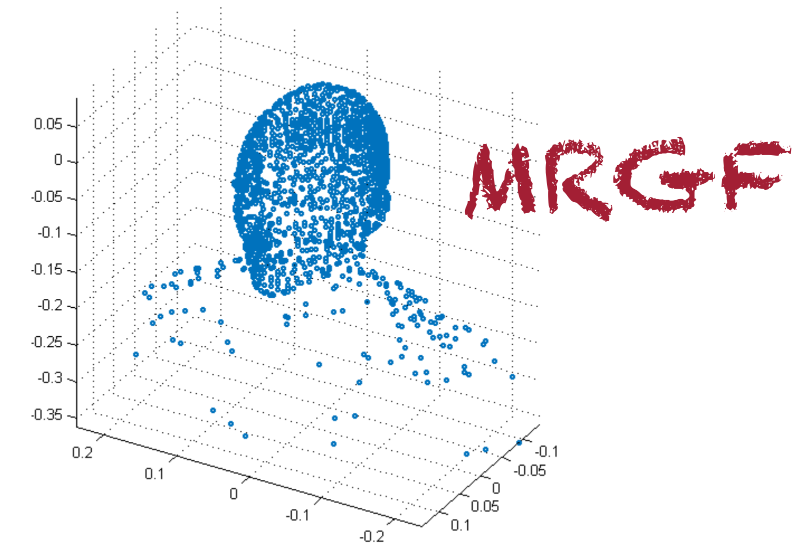
reuse for as many coils as desired
with fixed body model

Two-Stage approach

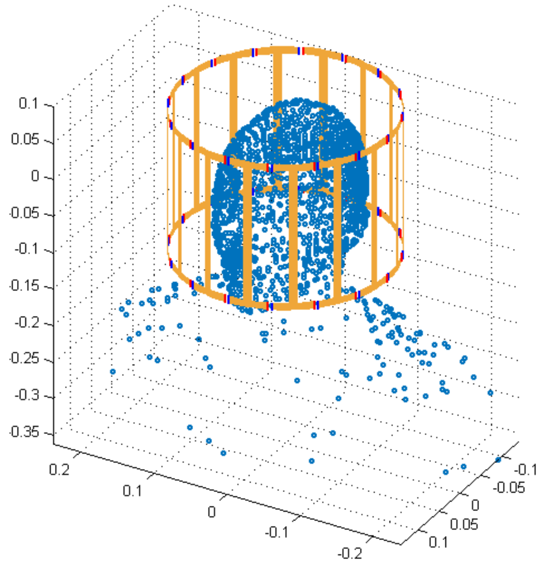
- **Accelerated Integral Equation solver for MRI coils**
  - **Off-line phase: MRGF pre-computation**



once per model



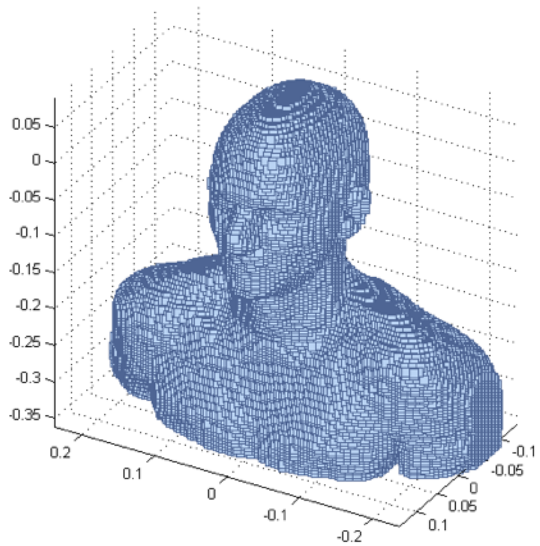
  - **On-line phase: MRGF use**



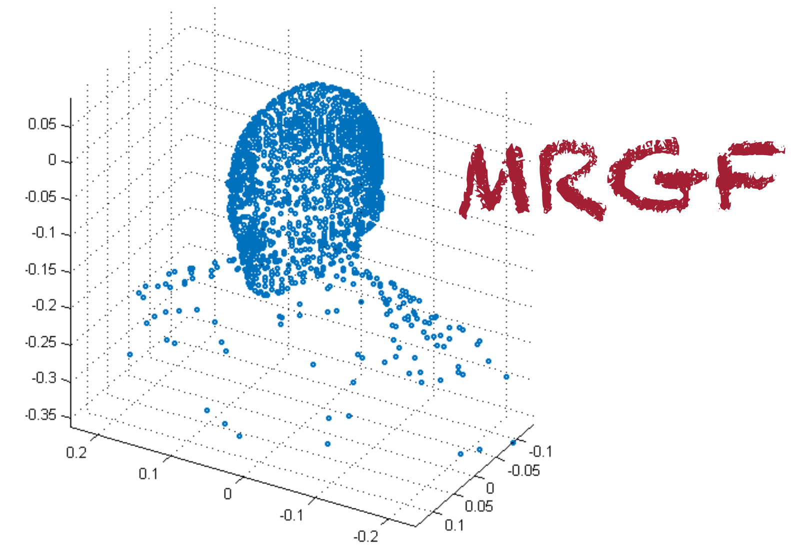
reuse for as many coils as desired
with fixed body model

Two-Stage approach

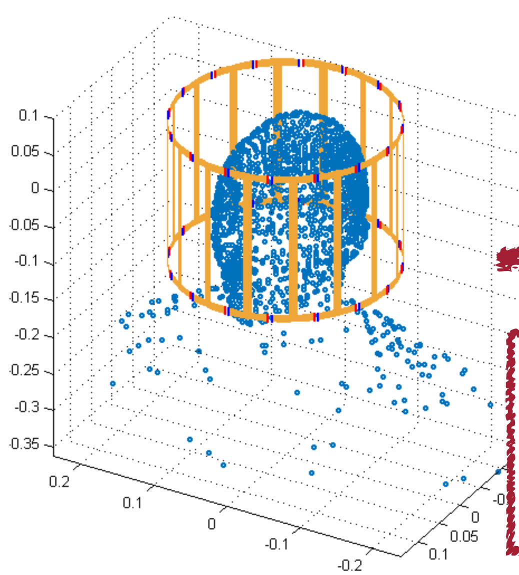
- **Accelerated Integral Equation solver for MRI coils**
  - **Off-line phase: MRGF pre-computation**



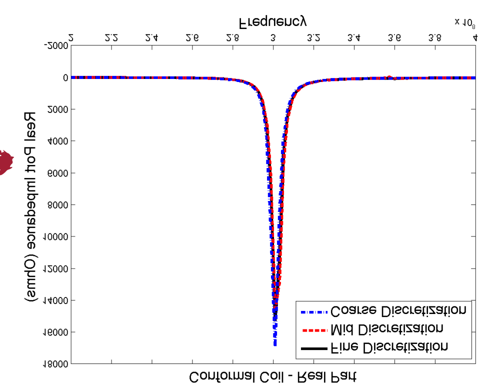
once per model



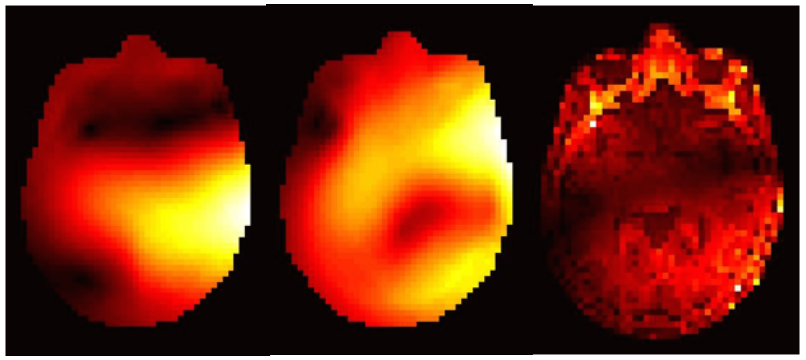
  - **On-line phase: MRGF use**



Accelerated full-wave EM solver



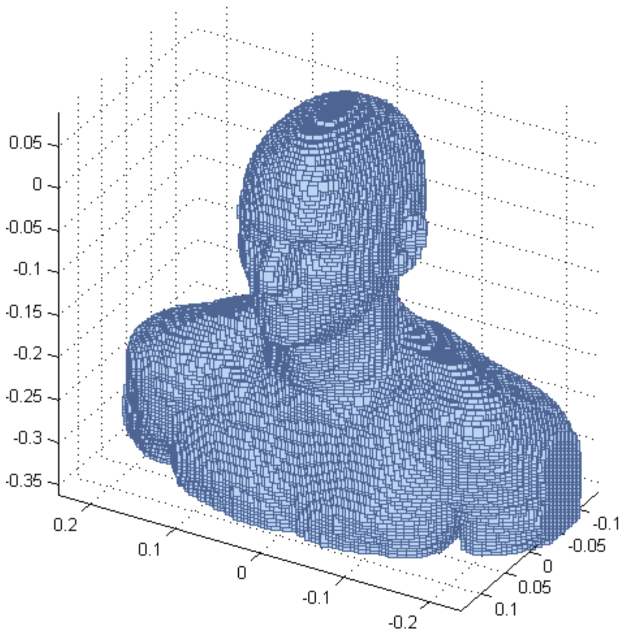
&



Detailed application on an realistic case

Pre-computation: equivalent model

- **Magnetic Resonance Green Function: Pre-computation**

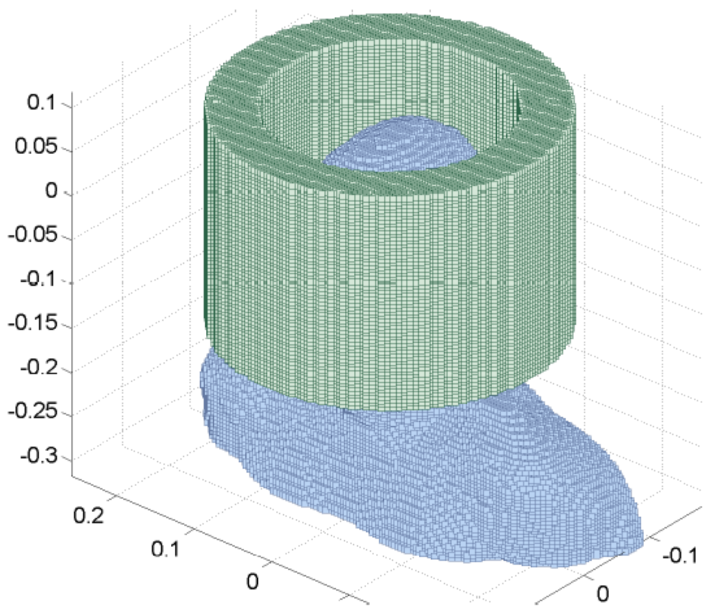


DUKE 4mm @7T

266,019 Voxels

Pre-computation: leverage knowledge

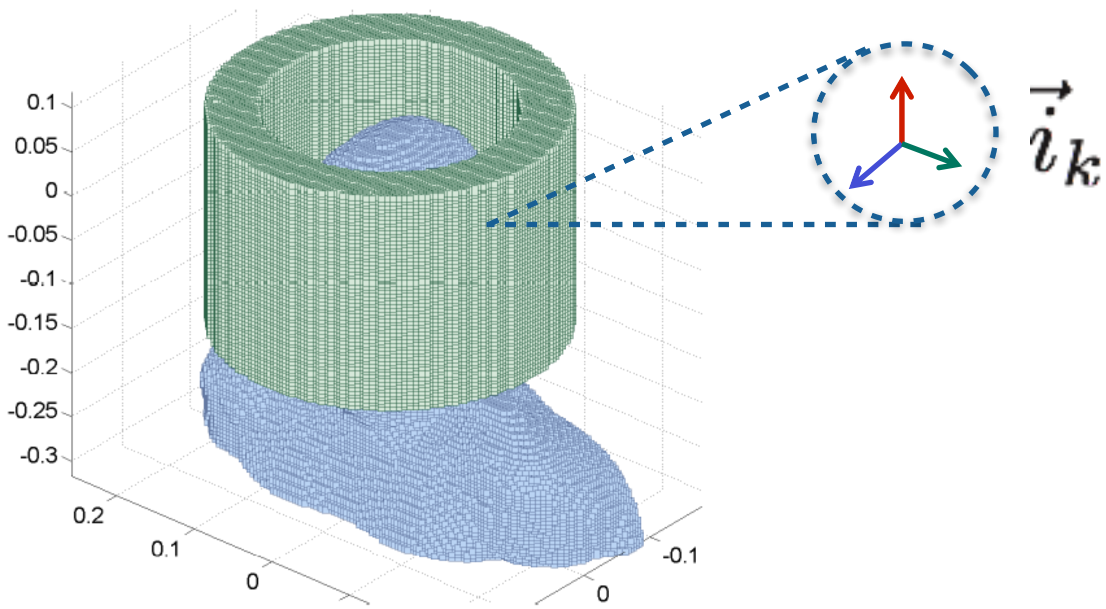
- **Magnetic Resonance Green Function: Pre-computation**
  - Coil domain



inner radius 13cm
outer radius 18cm
length 24cm

Pre-computation: generate any possible excitation

- **Magnetic Resonance Green Function: Pre-computation**
  - Generate the all possible incident fields



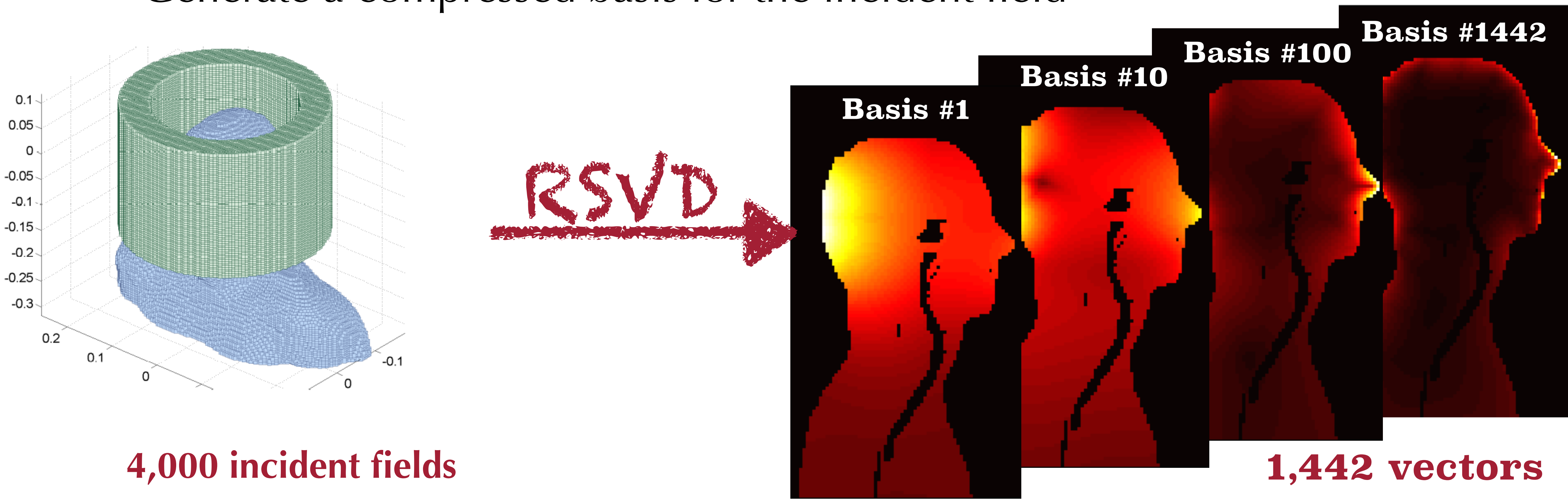
189,000 dipoles

3 components / dipole

4,000 random excitations

Pre-computation: basis for incident field

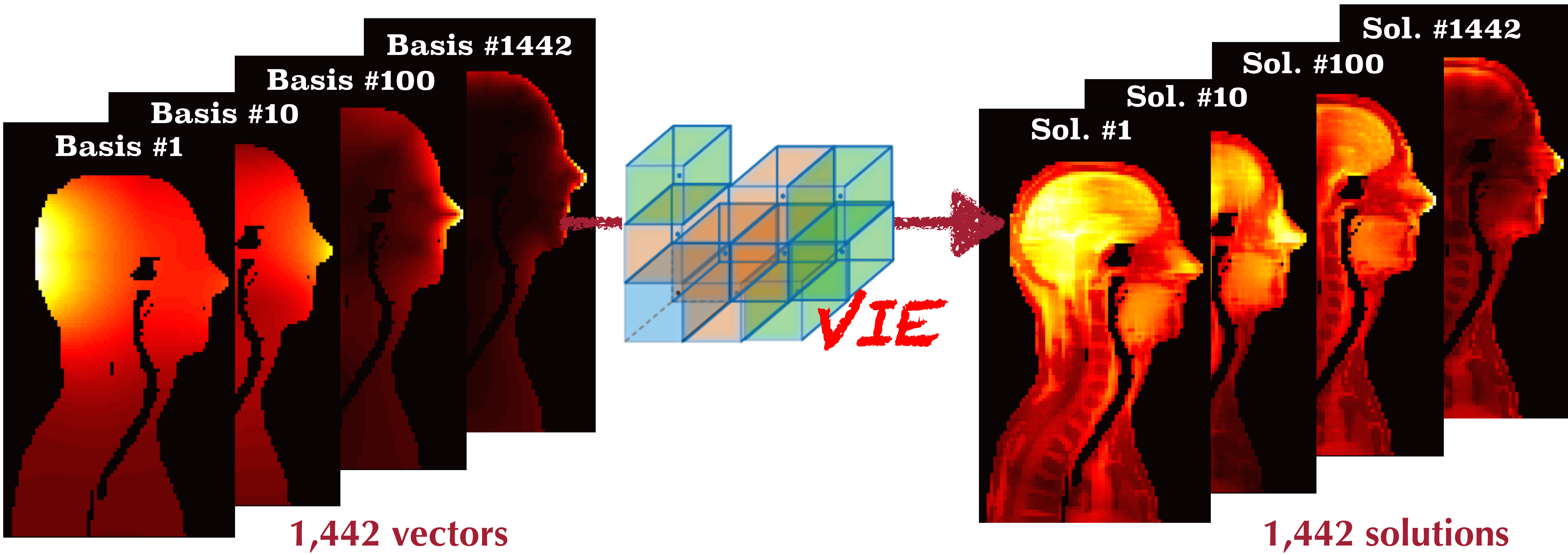
- **Magnetic Resonance Green Function: Pre-computation**
  - Generate a compressed basis for the incident field



[5] **Halko et al**, "Finding structure with randomness: probabilistic algorithms for constructing approximate matrix decompositions," **SIAM Rev. 2011.**

Pre-computation: solve scattering problem

- **Magnetic Resonance Green Function: Pre-computation**
  - Solve for each vector of the basis

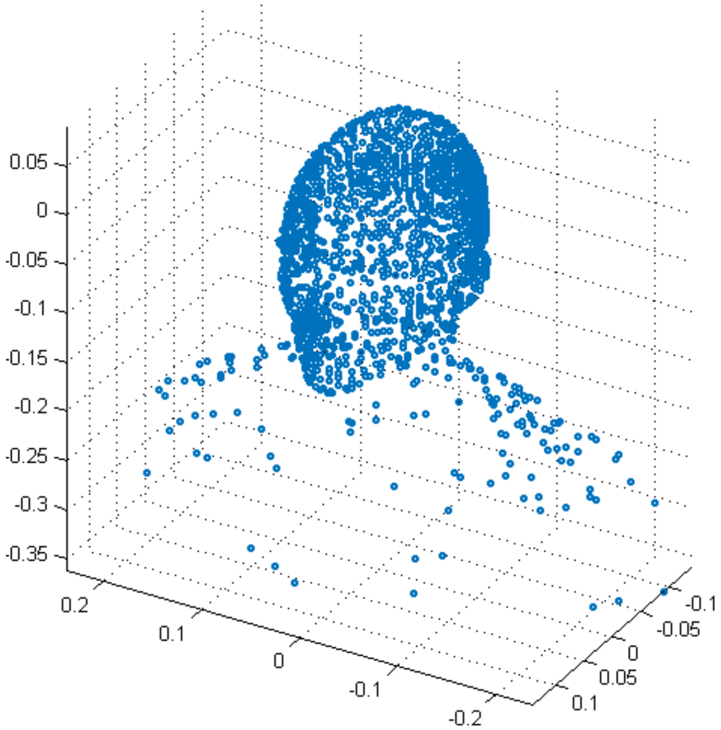


[4] **Polimeridis et al.**, "Stable FFT-JVIE solvers for fast analysis of highly inhomogeneous dielectric objects", **JCP 2014**

Pre-computation: select interpolation points

- **Magnetic Resonance Green Function: Pre-computation**
  - Generate a reduced set of interpolation points

1,442 vectors



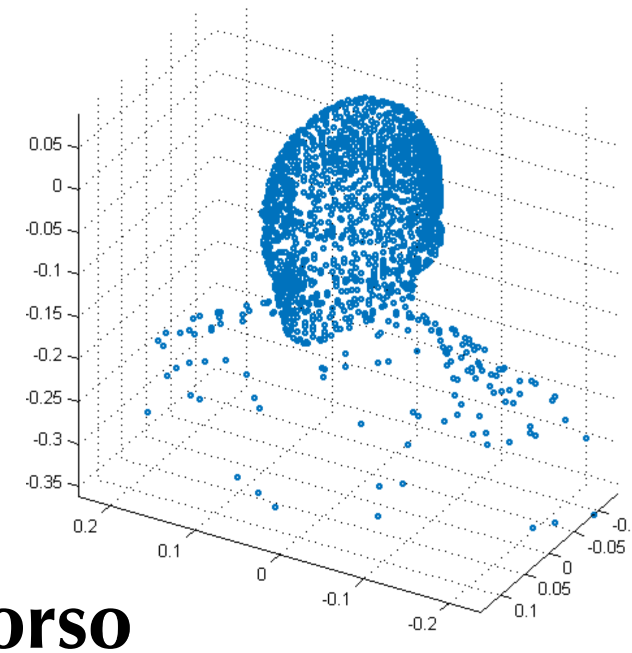
1,609 interpolation points

[6] **Chaturantabut** et al, "Nonlinear model reduction via discrete empirical interpolation," ***SIAM J. Sci. Comput.* 2010.**

Pre-computation: results

- **Magnetic Resonance Green Function**
  - Set of interpolation points in the body
  - Some pre-computed matrices

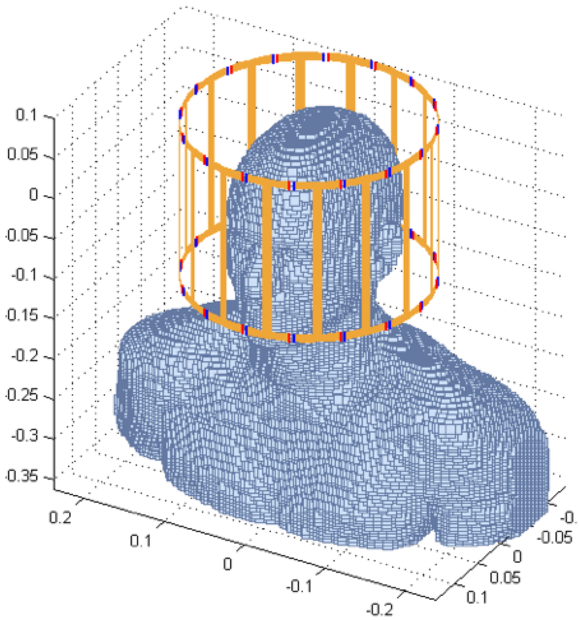
MRGF



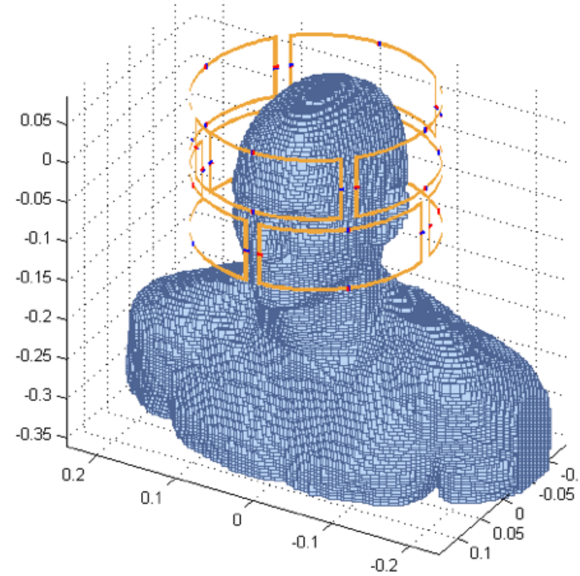
- **For 4mm DUKE 7T, head and torso**
  - **1609 DEIM points** (initially 266019 voxels)
  - **1442 basis vectors** (from 4000 excitations on 189000 dipoles)
  - Elapsed time 31 h 32 min (ONE TIME for a given model)
  - ~16GB storage

Non-Accelerated approach

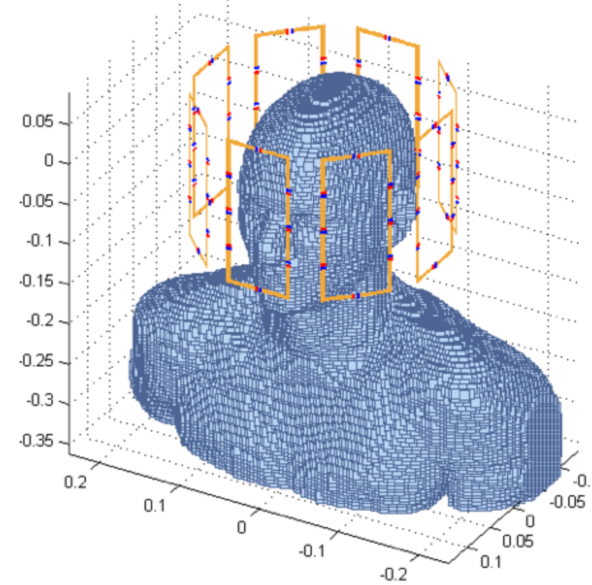
Full wave EM
Integral Equation Solver



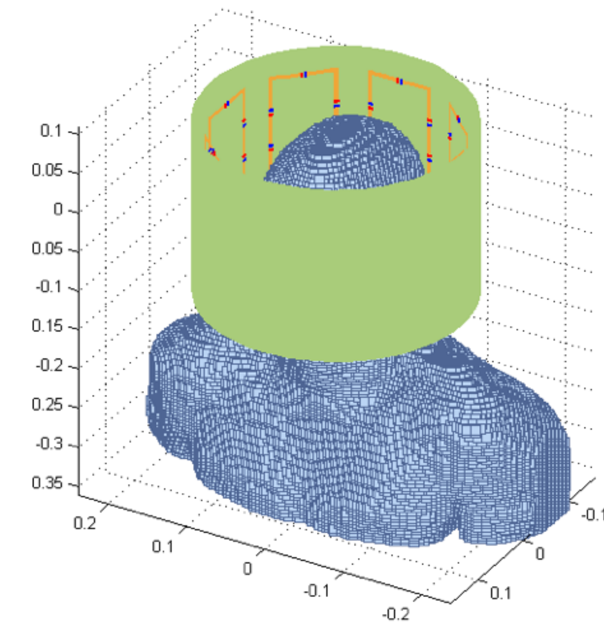
~8h29m overall
(~18 min/port)



~7h30m overall
(~17 min/port)



~9h32m overall
(~15 min/port)

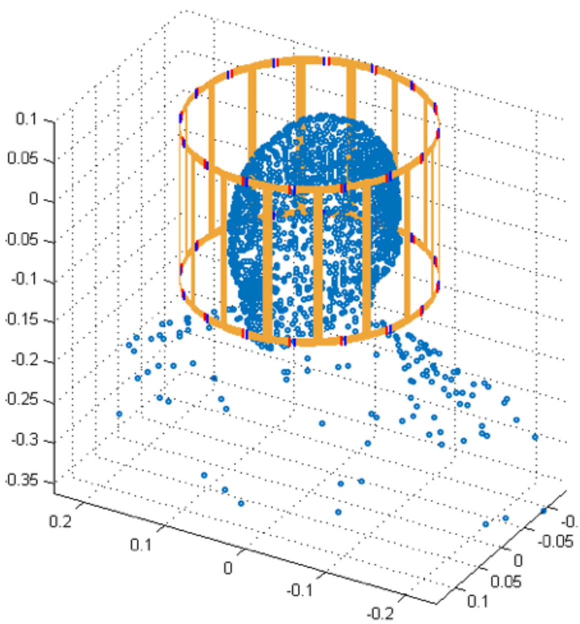


~17h13m overall
(~30 min/port)

MRGF-based acceleration

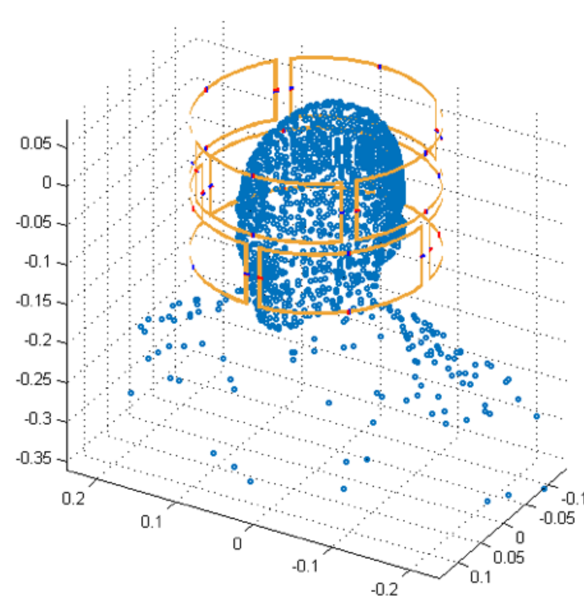
MRGF

Accelerated Full wave EM Integral Equation Solver



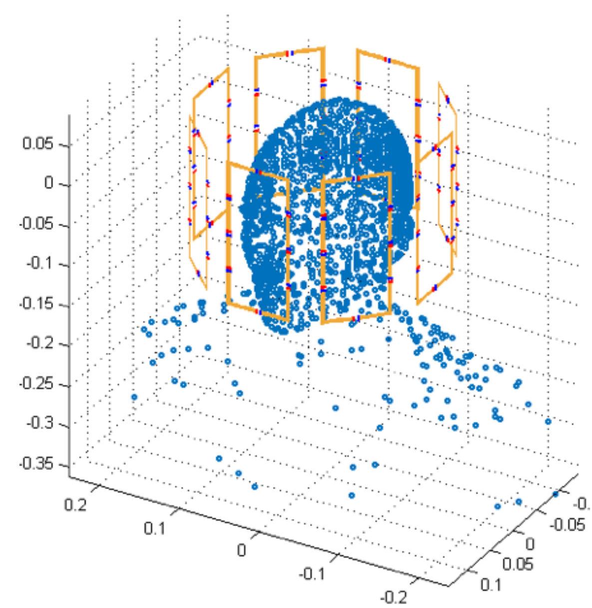
~8h29m overall
(~18 min/port)

~1min overall



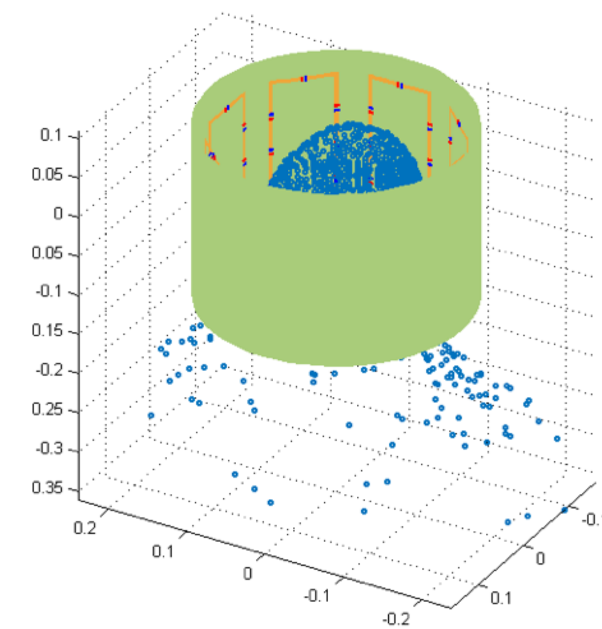
~7h30m overall
(~17 min/port)

~2min overall



~9h32m overall
(~15 min/port)

~2min overall



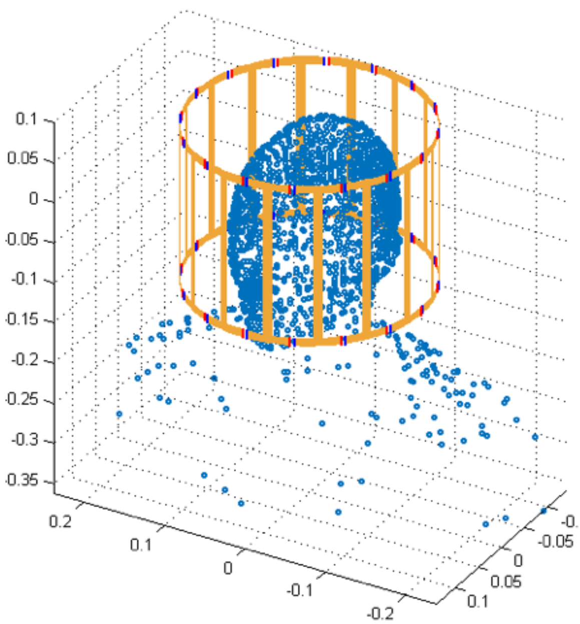
~17h13m overall
(~30 min/port)

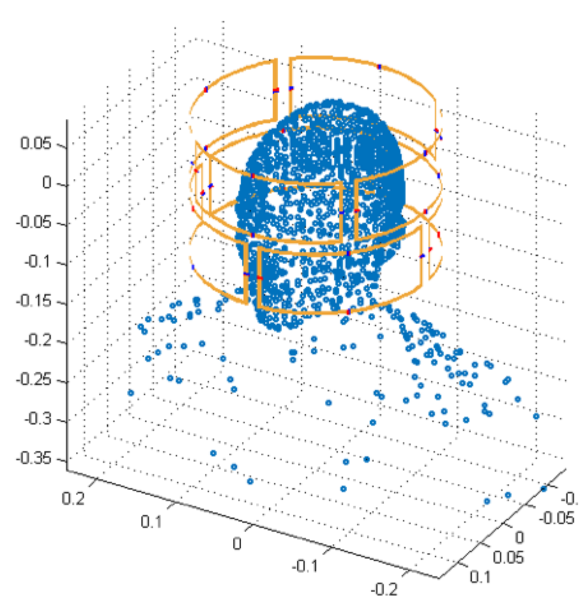
~7min overall

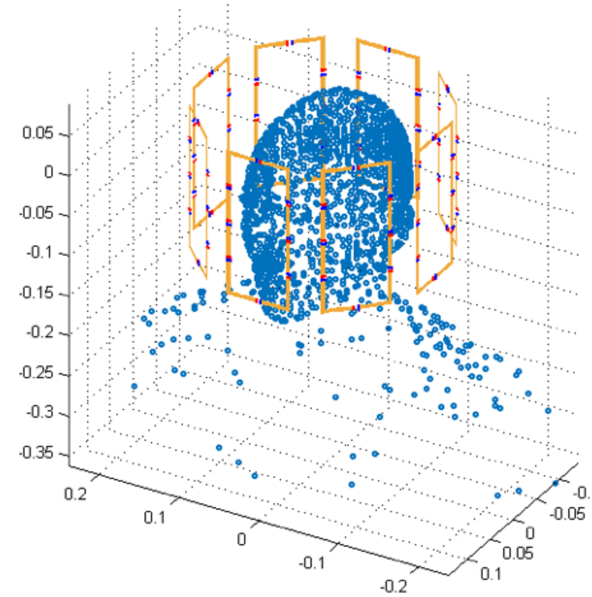
MRGF-based acceleration

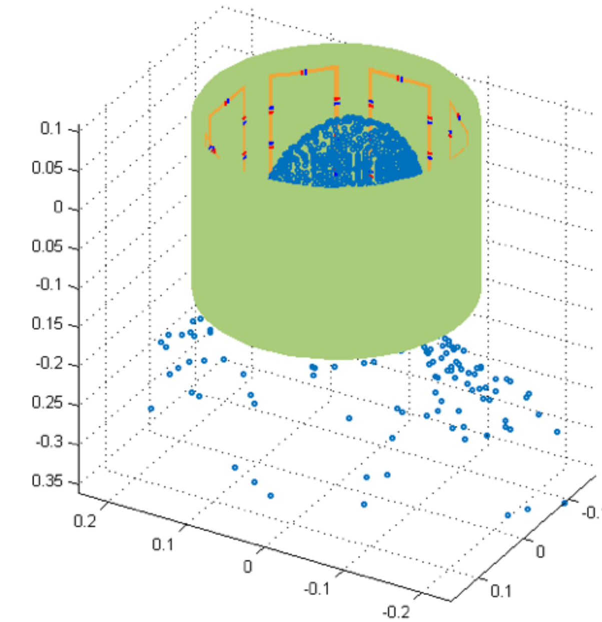
MRGF

Accelerated Full wave EM Integral Equation Solver







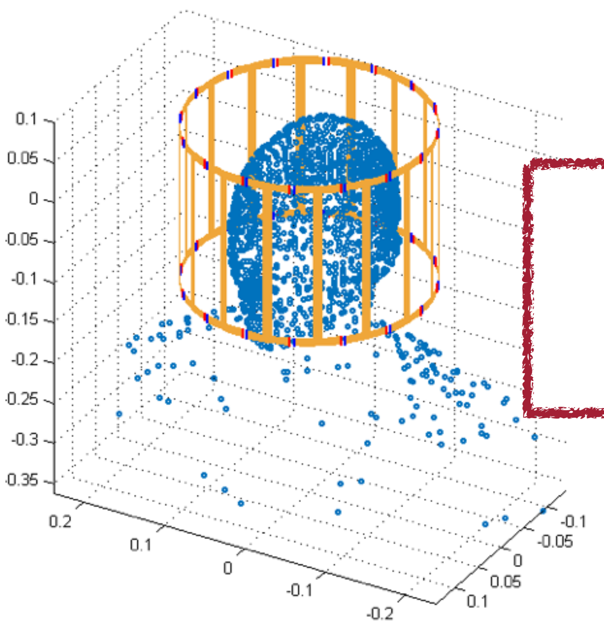


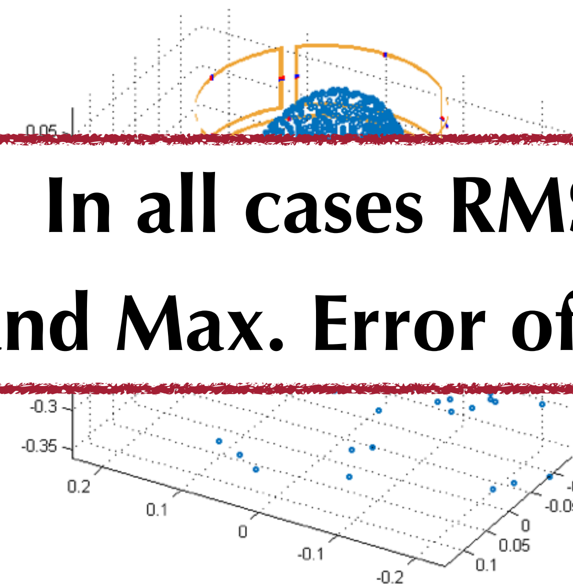
~8h29m overall (~18 min/port)	~7h30m overall (~17 min/port)	~9h32m overall (~15 min/port)	~17h13m overall (~30 min/port)
~462X	**~230X**	**~298X**	**~154X**
~1min overall	**~2min overall**	**~2min overall**	**~7min overall**

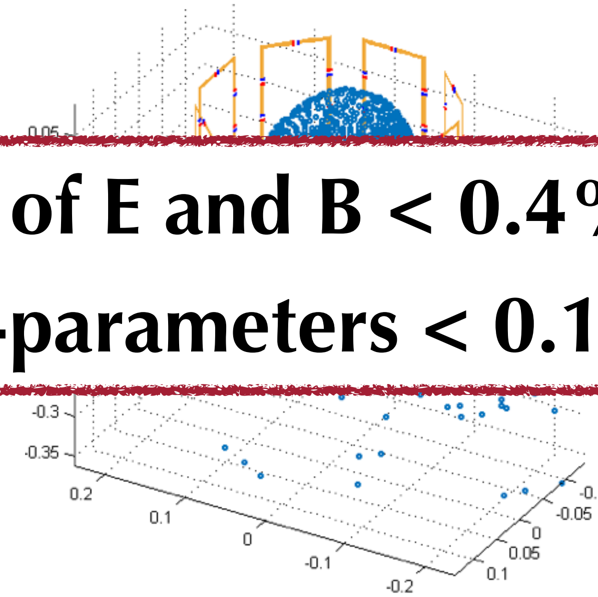
MRGF-based acceleration

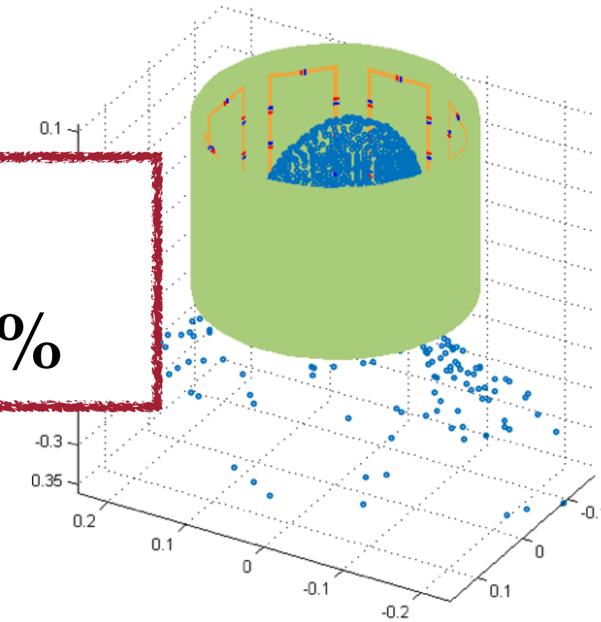
MRGF

Accelerated Full wave EM Integral Equation Solver









In all cases RMSE of E and B < 0.4% and Max. Error of S-parameters < 0.17%

~8h29m overall (~18 min/port)	~7h30m overall (~17 min/port)	~9h32m overall (~15 min/port)	~17h13m overall (~30 min/port)
~462X	**~230X**	**~298X**	**~154X**
~1min overall	**~2min overall**	**~2min overall**	**~7min overall**

To summarize…

- **By applying MRI customized simulation tools**
  - Domain decomposition and Integral Equation solvers
  - Slow **Off-Line** stage model pre-computation
    - Can be done for multiple models and frequencies
  - Fast **On-Line** stage model use
- **we can analyze a wide variety of coil array designs**
  - S-parameter matrix (with body)
  - Body E and B field distribution
  - **within minutes**
- **Enable technology for optimal coil design?**

Some references and Support

- **J. Fernández Villena et al,**
  "Fast Electromagnetic Analysis of MRI Transmit RF Coils based on Accelerated Integral Equation Methods",
  submitted to Physics in Medicine and Biology.

- **A. Hochman et al,**
  "Reduced-Order Models for Electromagnetic Scattering Problems",
  IEEE Transactions on Antennas and Propagation, 2014

- **A. Polimeridis et al,**
  "Stable FFT-JVIE solvers for fast analysis of highly inhomogeneous dielectric objects",
  Journal of Computational Physics, 2014.

- **SUPPORT:**
  - MIT Skoltech initiative
  - National Institute of Health
  - National Swiss Foundation

- **Contact:** jvillena@mit.edu http://web.mit.edu/jvillena/www