SAPOR-based MOR schemes with quadratic frequency dependence of excitation for FEM macromodels

Grzegorz Fotyga

Gdansk University of Technology, ETI
Department of Microwave and Antenna Engineering
Gdansk, Poland

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Presentation

1. Introduction
2. Model Order Reduction
3. Numerical experiments
4. Error Estimator
5. Conclusions

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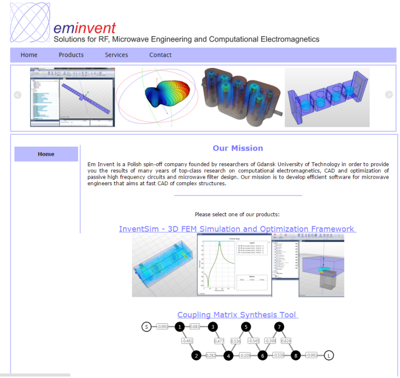
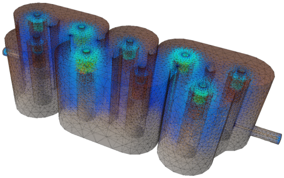
Introduction (1)

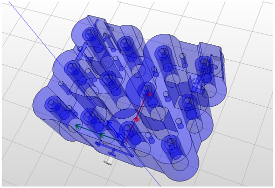
- FEM CAD EMinvent (http://www.eminvent.com/)

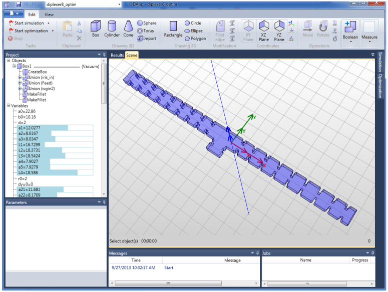


Introduction (2)

- FEM CAD EMinvent (http://www.eminvent.com/)

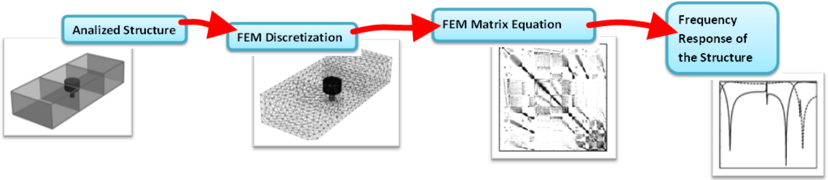






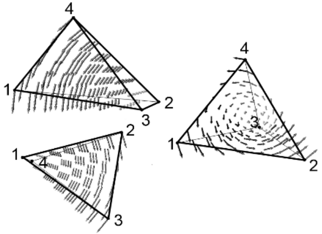
Introduction (3) - FEM

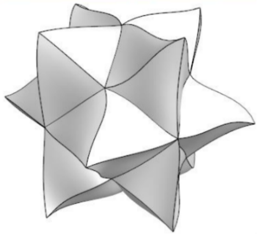
- The scheme of the FEM procedure



Introduction (4) FEM basis functions

- 2nd./ 3rd. order basis functions
- Curvilinear elements





Space	Basis functions	Assoc.
$\tilde{\mathcal{V}}_1$	ϕ_i	$\{i\}$
$\tilde{\mathcal{V}}_2$	$\phi_i\phi_j$	$\{ij\}$
$\tilde{\mathcal{V}}_3$	$\phi_i\phi_j(\phi_i - \phi_j)$,	$\{ij\}$
	$\phi_i\phi_j\phi_k$	$\{ijk\}$

Space	Basis functions	Assoc.
$\tilde{\mathcal{A}}_1$	$\phi_i\nabla\phi_j - \phi_j\nabla\phi_i$	$\{ij\}$
$\tilde{\mathcal{A}}_2$	$3\phi_j\phi_k\nabla\phi_i - \nabla(\phi_i\phi_j\phi_k)$,	$\{ijk\}$
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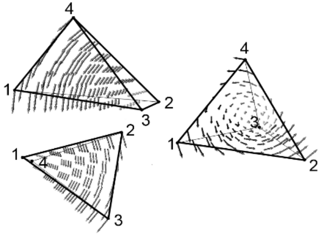

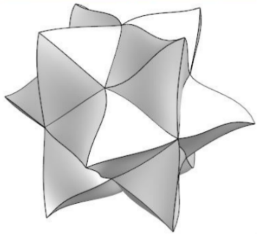

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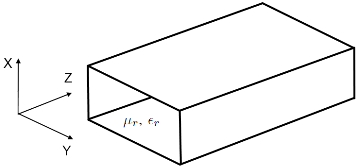
Introduction (5) FEM

- Vectorial wave equation:

$$\nabla \times \frac{1}{\mu_r} \nabla \times \vec{E} - k_0^2 \epsilon_r \vec{E} = 0.$$

- The equivalent FEM matrix equation is obtained by discretizing the computational domain using a tetrahedral mesh and applying Galerkin method:

$$(\mathbf{K} - k^2 \mathbf{M}) \cdot \mathbf{x}(k) = \mathbf{b}(k).$$



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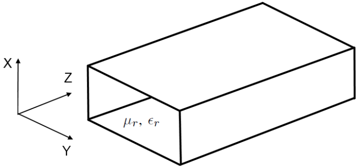

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- **K** and **M** (stiffness and mass matrices respectively) are $N \times N$ real, symmetric and sparse,
- **b** is the excitation vector,
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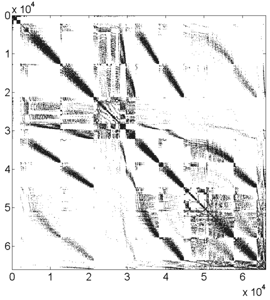
- $k = 2\pi/\lambda$ is the wavenumber.



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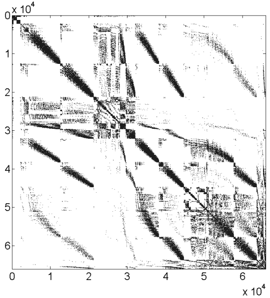

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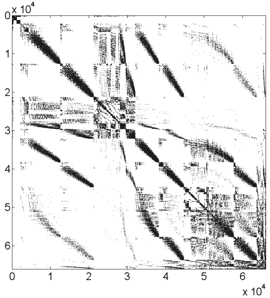

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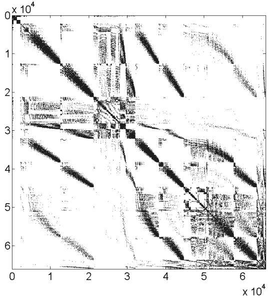

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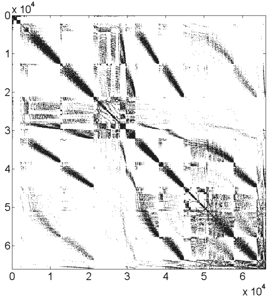
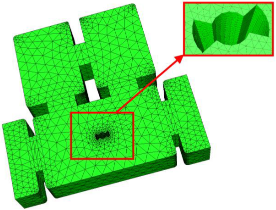
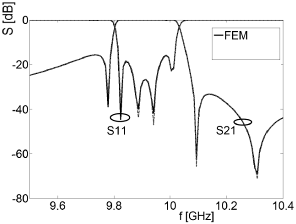


Introduction (6) - Numerical Example

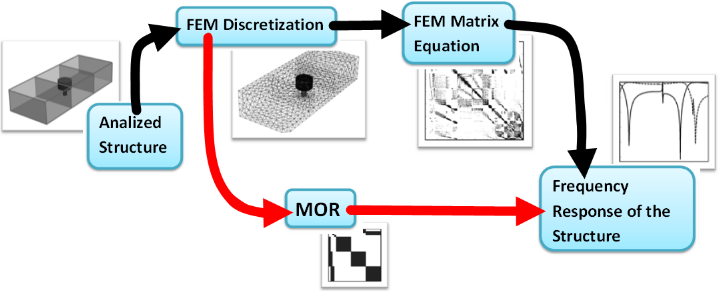
- Number of variables: ca. 440.000
- Number of freqency points: 101
- Solution time: ca. 2h
- i7, 16GB RAM





Goals (1)

- The scheme of the FEM-MOR procedure.



- Y. Zhu and A. C. Cangellaris, "Macro-elements for efficient FEM simulation of small geometric features in waveguide components," IEEE Trans. Microw. Theory Tech., vol. 48, no. 12, pp. 2254-2260, Dec. 2000.
- V. de la Rubia and J. Zapata, "Microwave circuit design by means of direct decomposition in the finite-element method," IEEE Trans. Microw. Theory Tech., vol. 55, no. 7, pp. 1520-1530, Jul. 2007.
- Lee, Shih-hao, "Efficient Finite Element Electromagnetic Analysis for High-Frequency/High-Speed Circuits And Multiconductor Transmission Lines", PhD Thesis, Director of Research: Jin, Jianming

Goals (2) - Features of the MOR

Features of the MOR:

- realiable
- automatic
- frequency independent
- suitable to optimization schemes

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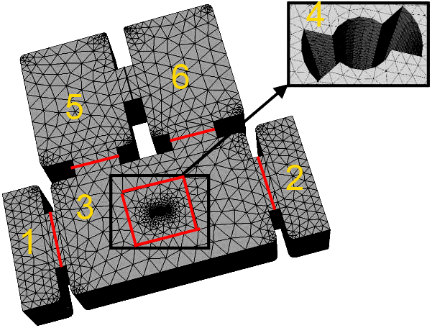
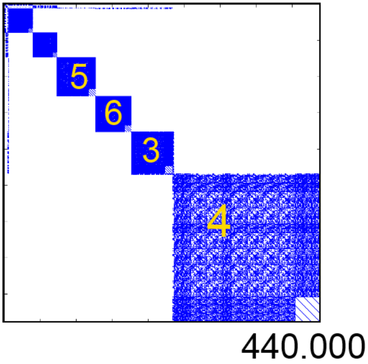
MOR

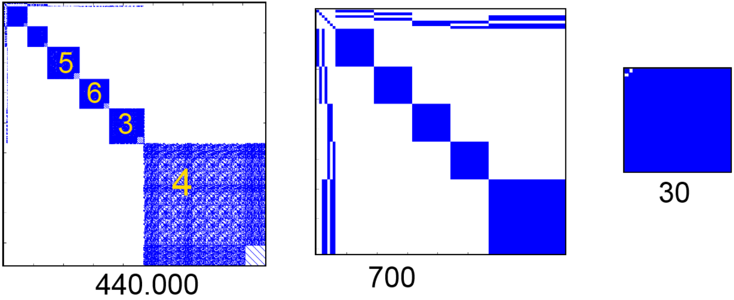
- Dividing the computational domain





MOR (1) - number of variables

- Original FEM matrix
- After 1st reduction
- After 2nd reduction - matrix is 15.000 times smaller!



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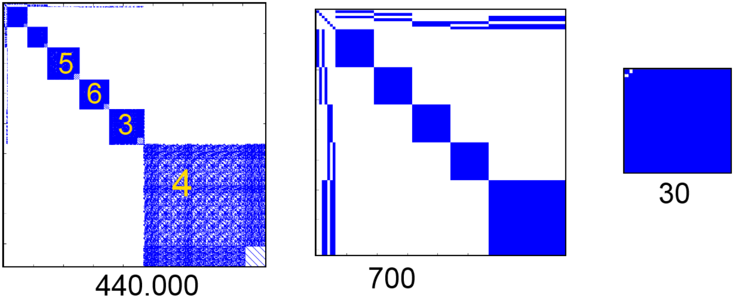

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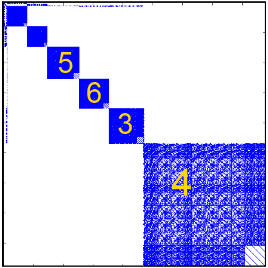
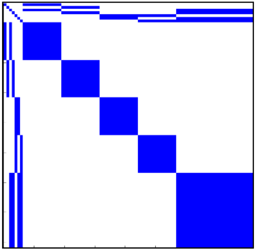
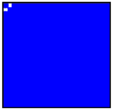
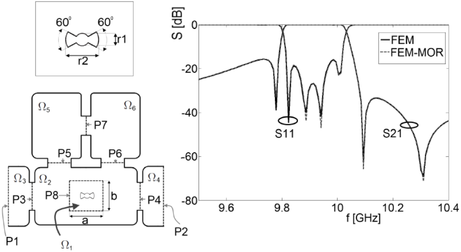






MOR (2)

- Amost the same S-characteristics obtained by means of original FEM and FEM-MOR.



(a) (b)

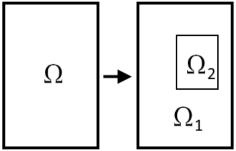
MOR (3)

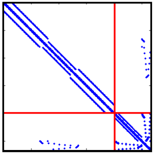
$$(\mathbf{K} - k^2\mathbf{M}) \cdot \mathbf{x}(k) = \mathbf{b}(k)$$

- Subregion Ω_1 is separated from the outer region Ω_2.
- Due to the separation of the subdomains the FEM matrix equation splits into:

$$\left(\begin{bmatrix} \mathbf{K}_1 & \mathbf{S}_K^T \\ \mathbf{S}_K & \mathbf{K}_2 \end{bmatrix} - k^2 \begin{bmatrix} \mathbf{M}_1 & \mathbf{S}_M^T \\ \mathbf{S}_M & \mathbf{M}_2 \end{bmatrix} \right) \cdot \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{b} \\ \mathbf{0} \end{bmatrix},$$

- The mesh edges that couple two separated subregions form four coupling matrices: $\mathbf{S}_K$, $\mathbf{S}_M$, $\mathbf{S}_K^T$ and $\mathbf{S}_M^T$.





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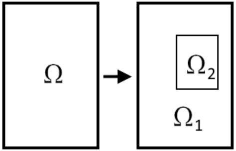

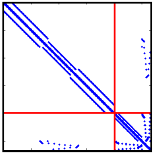

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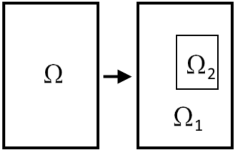

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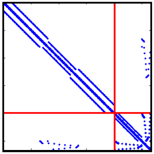

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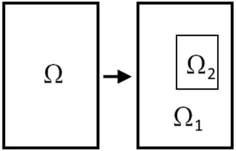

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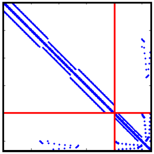

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MOR (4)

- The number of variables in the separated region Ω_2 is $\mathbf{N}_2$
- The number of variables in the outer region Ω_1 is $\mathbf{N}_1$
- $\mathbf{N}_2$ can be reduced by means of the REDUCTION algorithm
- input:
  - Matrices: $\mathbf{K}_2$, $\mathbf{M}_2$, $\mathbf{S}_K$ and $\mathbf{S}_M$
  - q - the reduction order
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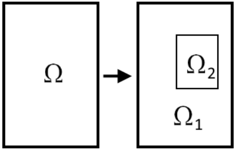

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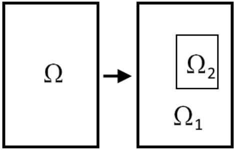

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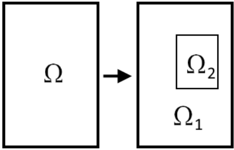

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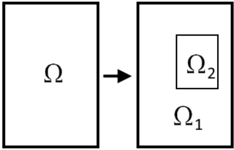

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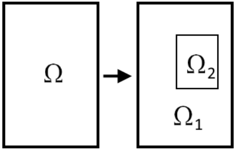

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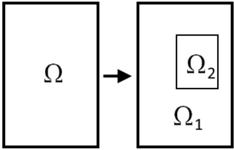

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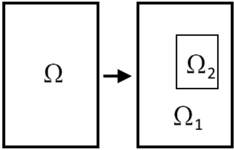

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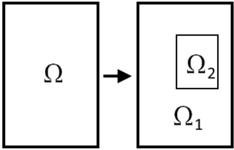


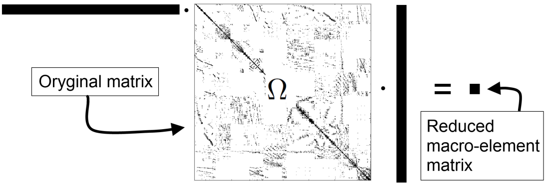
MOR (5)

- Matrices $\mathbf{K}_2$, $\mathbf{M}_2$, $\mathbf{S}_K$ and $\mathbf{S}_M$ from the FEM equation:

$$\left(\begin{bmatrix} \mathbf{K}_1 & \mathbf{S}_K^T \\ \mathbf{S}_K & \mathbf{K}_2 \end{bmatrix} - k^2 \begin{bmatrix} \mathbf{M}_1 & \mathbf{S}_M^T \\ \mathbf{S}_M & \mathbf{M}_2 \end{bmatrix}\right) \cdot \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{b} \\ \mathbf{0} \end{bmatrix},$$

- can be reduced using the orthogonal basis $\mathbf{V}$:

$$\left(\begin{bmatrix} \mathbf{K}_1 & \mathbf{S}_K^T \cdot \mathbf{V} \\ \mathbf{V}^T \cdot \mathbf{S}_K & \mathbf{V}^T \cdot \mathbf{K}_2 \cdot \mathbf{V} \end{bmatrix} - k^2 \begin{bmatrix} \mathbf{M}_1 & \mathbf{S}_M^T \cdot \mathbf{V} \\ \mathbf{V}^T \cdot \mathbf{S}_M & \mathbf{V}^T \cdot \mathbf{M}_2 \cdot \mathbf{V} \end{bmatrix}\right) \cdot \begin{bmatrix} \mathbf{x}_1 \\ \widetilde{\mathbf{x}}_2 \end{bmatrix} = \begin{bmatrix} \mathbf{b} \\ \mathbf{0} \end{bmatrix},$$



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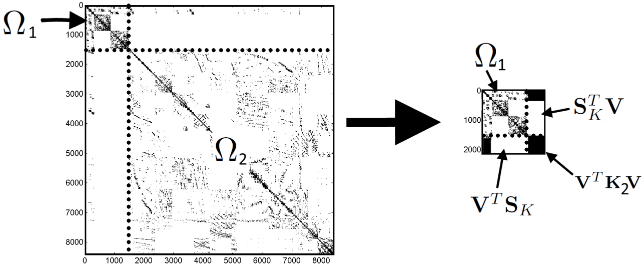
MOR (6)

- System matrix after MOR:



- The reduced region is called a macro-element.
- The number of variables in macro-element is $\mathbf{N}_M$.
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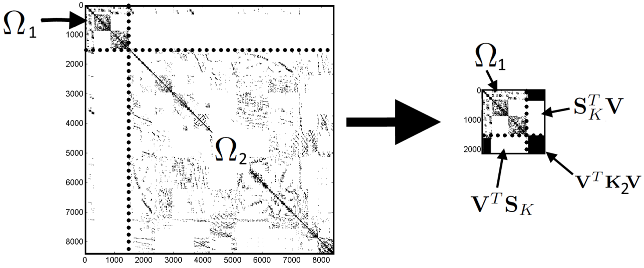

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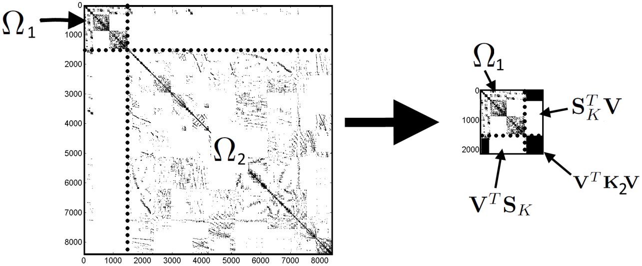

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MOR (7)

- The equation:

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- can be represented as:

$$\left(\mathbf{K}_1 - k^2\mathbf{M}_1\right) \cdot \mathbf{x}_1 + \left(\mathbf{S}_K^T - k^2\mathbf{S}_M^T\right) \cdot \mathbf{x}_2 = \mathbf{b}$$

$$\left(\mathbf{K}_2 - k^2\mathbf{M}_2\right) \cdot \mathbf{x}_2 = -\left(\mathbf{S}_K - k^2\mathbf{S}_M\right) \cdot \mathbf{x}_1.$$

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$$\mathbf{x}_2 = \mathbf{H}(k)\mathbf{x}_1,$$

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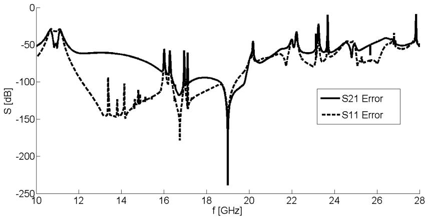
MOR (8)

- Reduction algorithms: ENOR, SAPOR $\Rightarrow$ **V**
- Error is low only for the center frequency!



- $(\mathbf{K} - k_0^2 M) \cdot \mathbf{x} = jk_0 \mathbf{b}$
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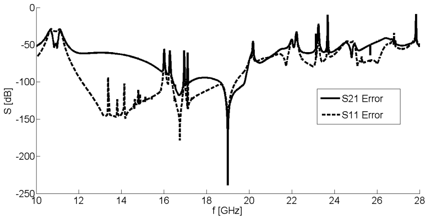

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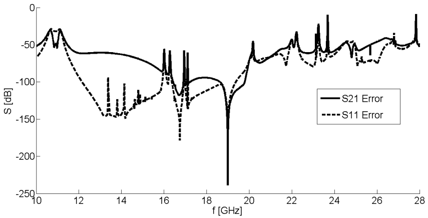

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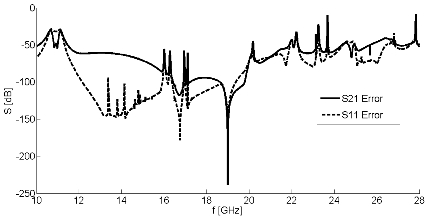

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- Shifting the equation:

$$\mathbf{H}(k) = \left(\mathbf{K}_2 - k^2\mathbf{M}_2\right)^{-1} \cdot \left(-\mathbf{S}_K + k^2\mathbf{S}_M\right).$$

- with $k = k_0 + \sigma$, we obtain:

$$\left(\sigma^2\mathbf{C} + \sigma\mathbf{D} + \mathbf{K}\right) \cdot \mathbf{H}(\sigma) = \sigma^2\mathbf{B}_2 + \sigma\mathbf{B}_1 + \mathbf{B}_0,$$

- where:

$$\mathbf{C} = -\mathbf{M}_2; \quad \mathbf{D} = -2k_0\mathbf{M}_2; \quad \mathbf{K} = \mathbf{K}_2 - k_0^2\mathbf{M}_2;$$

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MOR (10)

- Introducing a new matrix: $\mathbf{H}_Z$

$$\sigma \mathbf{C}\mathbf{H} + \mathbf{H}_z = \mathbf{B}_2 \sigma + \mathbf{B}_1$$

- we obtain:

$$(\mathbf{I} - \sigma \mathbf{A}) \begin{bmatrix} \mathbf{H} \\ \mathbf{H}_z \end{bmatrix} = \begin{bmatrix} \mathbf{K}^{-1}\mathbf{B}_0 \\ \mathbf{B}_1 \end{bmatrix} + \sigma \begin{bmatrix} 0 \\ \mathbf{B}_2 \end{bmatrix},$$

- where:

$$\mathbf{A} = \begin{bmatrix} \mathbf{K}^{-1}\mathbf{D} & \mathbf{K}^{-1} \\ \mathbf{C} & \mathbf{0} \end{bmatrix}.$$

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MOR (11)

- By moving $(\mathbf{I} - \sigma \mathbf{A})$ to the RHS and performing a Maclaurin series expansion, we obtain:

$$\left[\begin{array}{c} \mathbf{H} \\ \mathbf{H}_z \end{array}\right] = \left(\mathbf{I} + \sigma \mathbf{A} + \sigma^2 \mathbf{A}^2 + \sigma^3 \mathbf{A}^3 \ldots\right) \cdot \left(\left[\begin{array}{c} \mathbf{K}^{-1}\mathbf{B}_0 \\ \mathbf{B}_1 \end{array}\right] + \sigma \left[\begin{array}{c} 0 \\ \mathbf{B}_2 \end{array}\right]\right).$$

- The block moments of $\mathbf{H}$: $\mathbf{H}_0, \mathbf{H}_1, \mathbf{H}_2 \ldots$

$$\mathbf{H}_i = \left[\mathbf{A}^i \left[\begin{array}{c} 0 \\ \mathbf{B}_2 \end{array}\right], \; \mathbf{A}^i \left[\begin{array}{c} \mathbf{K}^{-1}\mathbf{B}_0 \\ \mathbf{B}_1 \end{array}\right]\right],$$

- The orhogonal basis $\mathbf{V}$ of order q is spanned by $\mathbf{H}_0, \mathbf{H}_1, \mathbf{H}_2 \ldots, \mathbf{H}_q$.

$$\mathbf{V}_q = \left[\mathbf{H}_1, \mathbf{H}_2, \mathbf{H}_3, \ldots \mathbf{H}_q\right].$$

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MOR (11)

- By moving $(\mathbf{I} - \sigma \mathbf{A})$ to the RHS and performing a Maclaurin series expansion, we obtain:

$$\begin{bmatrix} \mathbf{H} \\ \mathbf{H}_z \end{bmatrix} = \left(\mathbf{I} + \sigma \mathbf{A} + \sigma^2 \mathbf{A}^2 + \sigma^3 \mathbf{A}^3 \dots \right) \cdot \left(\begin{bmatrix} \mathbf{K}^{-1}\mathbf{B}_0 \\ \mathbf{B}_1 \end{bmatrix} + \sigma \begin{bmatrix} 0 \\ \mathbf{B}_2 \end{bmatrix} \right).$$

- The block moments of $\mathbf{H}$: $\mathbf{H}_0, \mathbf{H}_1, \mathbf{H}_2 \dots$

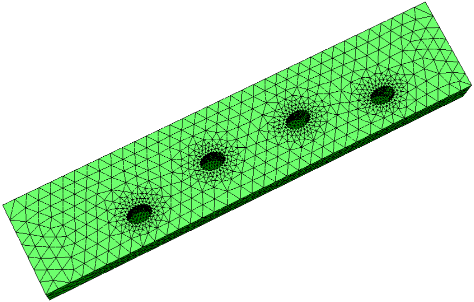
$$\mathbf{H}_i = \left[\mathbf{A}^i \begin{bmatrix} 0 \\ \mathbf{B}_2 \end{bmatrix}, \;\; \mathbf{A}^i \begin{bmatrix} \mathbf{K}^{-1}\mathbf{B}_0 \\ \mathbf{B}_1 \end{bmatrix} \right],$$

- The orhogonal basis $\mathbf{V}$ of order q is spanned by $\mathbf{H}_0, \mathbf{H}_1, \mathbf{H}_2 \dots, \mathbf{H}_q$.

$$\mathbf{V}_q = \left[\mathbf{H}_1, \mathbf{H}_2, \mathbf{H}_3, \dots \mathbf{H}_q \right].$$

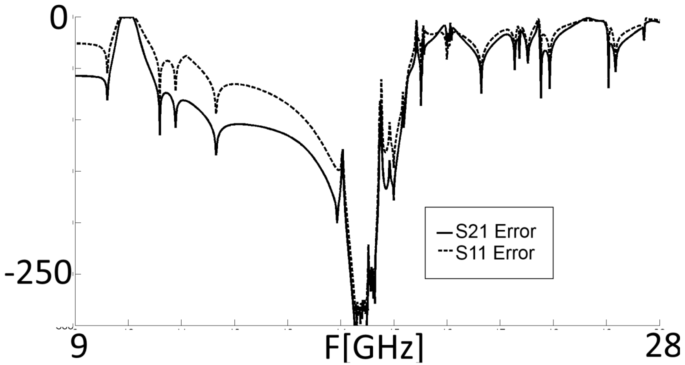
Numerical Experiment

- Waveguide filter with elliptic posts.



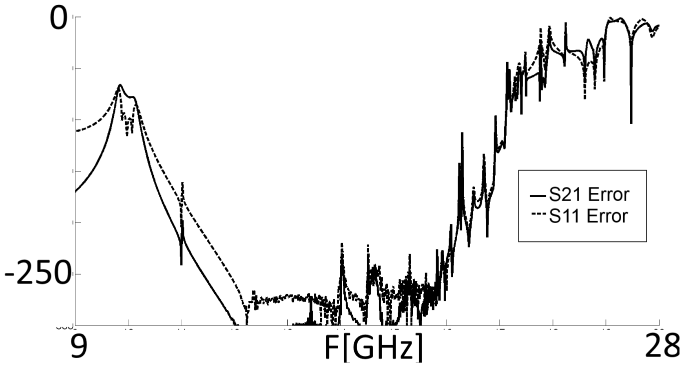
Numerical Experiment

- $q = 10$
- $20log(S_{MOR} - S_{FEM})$



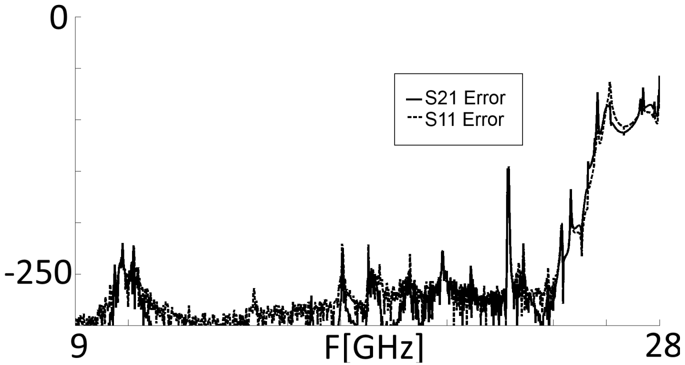
Numerical Experiment

- $q = 20$
- $20log(S_{MOR} - S_{FEM})$



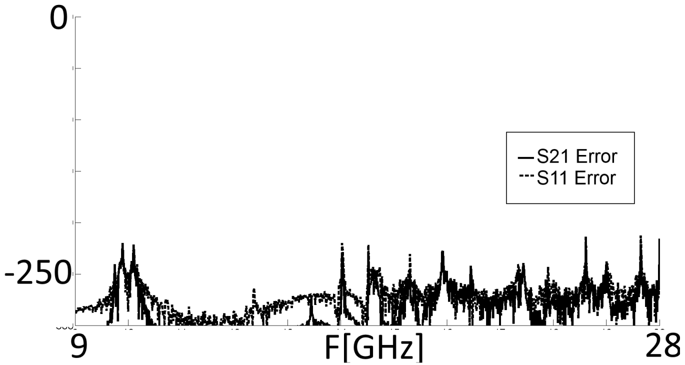
Numerical Experiment

- $q = 40$
- $20log(S_{MOR} - S_{FEM})$



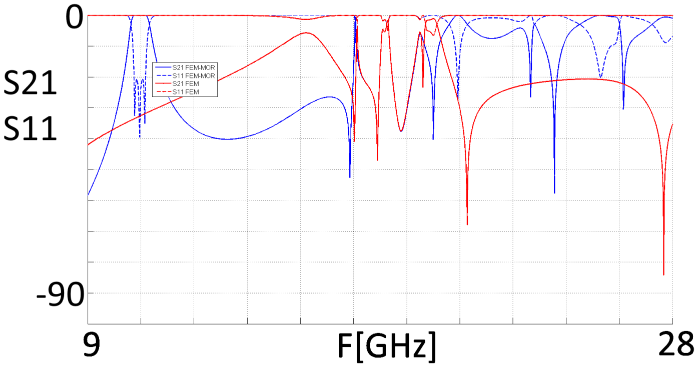
Numerical Experiment

- $q = 70$
- $20log(S_{MOR} - S_{FEM})$



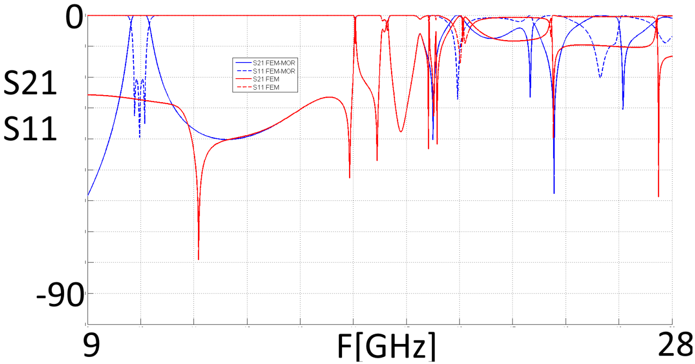
Numerical Experiment

- $q = 5$



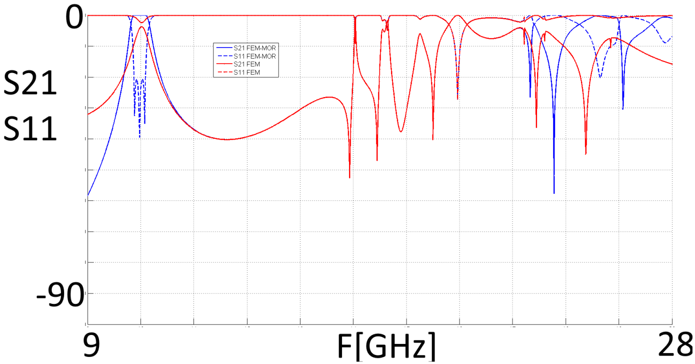
Numerical Experiment

- $q = 10$



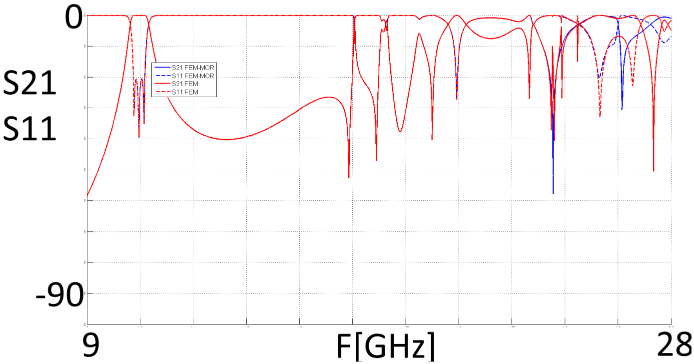
Numerical Experiment

- $q = 15$



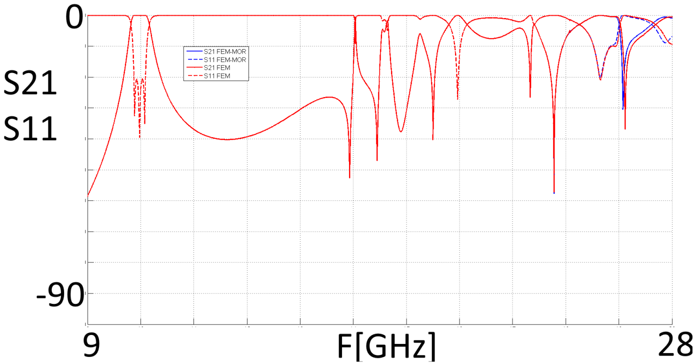
Numerical Experiment

- $q = 20$



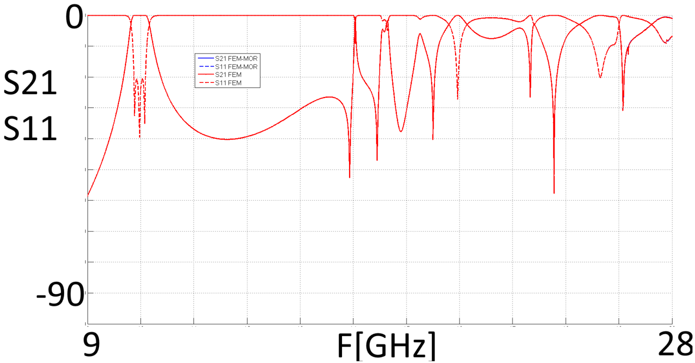
Numerical Experiment

- $q = 25$



Numerical Experiment

- $q = 30$



Error estimation (1)

- The equation for the separated domain

$$\left(\mathbf{K}_2 - k^2\mathbf{M}_2\right) \cdot \mathbf{x}_2 = -\left(\mathbf{S}_K - k^2\mathbf{S}_M\right) \cdot \mathbf{x}_1.$$

- Residuum

$$\mathbf{R}(k) = -\left(\mathbf{K}_2 - k^2\mathbf{M}_2\right) \cdot \mathbf{V} \cdot \widetilde{\mathbf{x}}_2 - \left(\mathbf{S}_K - k^2\mathbf{S}_M\right) \cdot \mathbf{x}_1.$$

- Error estimation = Residuum * RHS

$$err_{EST}(k) = ||\left(\mathbf{S}_K - k^2\mathbf{S}_M\right) \cdot \mathbf{R}(k)|| = ||k^4(\mathbf{S}_M, \mathbf{S}_M) - k^2(\mathbf{S}_M, \mathbf{S}_K) -$$
$$-k^2(\mathbf{S}_M, \mathbf{K}_2 \cdot \mathbf{V}) + k^4(\mathbf{S}_M, \mathbf{M}_2 \cdot \mathbf{V}) \cdot \widetilde{\mathbf{x}}_2 - k^2(\mathbf{S}_M, \mathbf{S}_K) +$$
$$(\mathbf{S}_K, \mathbf{S}_K) + (\mathbf{S}_K, \mathbf{K}_2 \cdot \mathbf{V}) \cdot \widetilde{\mathbf{x}}_2 - k^2(\mathbf{S}_K, \mathbf{M}_2 \cdot \mathbf{V}) \cdot \widetilde{\mathbf{x}}_2||,$$

Error estimation (1)

- The equation for the separated domain

$$\left(\mathbf{K}_2 - k^2\mathbf{M}_2\right)\cdot\mathbf{x}_2 = -\left(\mathbf{S}_K - k^2\mathbf{S}_M\right)\cdot\mathbf{x}_1.$$

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Error estimation (1)

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$$\left(\mathbf{K}_2 - k^2\mathbf{M}_2\right)\cdot\mathbf{x}_2 = -\left(\mathbf{S}_K - k^2\mathbf{S}_M\right)\cdot\mathbf{x}_1.$$

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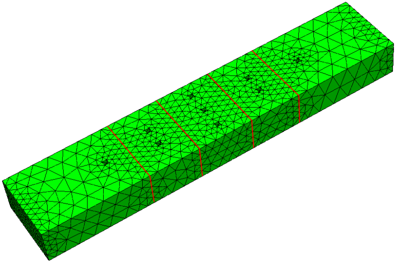
$$err_{EST}(k) = ||\left(\mathbf{S}_K - k^2\mathbf{S}_M\right)\cdot\mathbf{R}(k)|| = ||k^4(\mathbf{S}_M,\mathbf{S}_M) - k^2(\mathbf{S}_M,\mathbf{S}_K) - \\ -k^2(\mathbf{S}_M,\mathbf{K}_2\cdot\mathbf{V}) + k^4(\mathbf{S}_M,\mathbf{M}_2\cdot\mathbf{V})\cdot\widetilde{\mathbf{x}}_2 - k^2(\mathbf{S}_M,\mathbf{S}_K) + \\ (\mathbf{S}_K,\mathbf{S}_K) + (\mathbf{S}_K,\mathbf{K}_2\cdot\mathbf{V})\cdot\widetilde{\mathbf{x}}_2 - k^2(\mathbf{S}_K,\mathbf{M}_2\cdot\mathbf{V})\cdot\widetilde{\mathbf{x}}_2||,$$

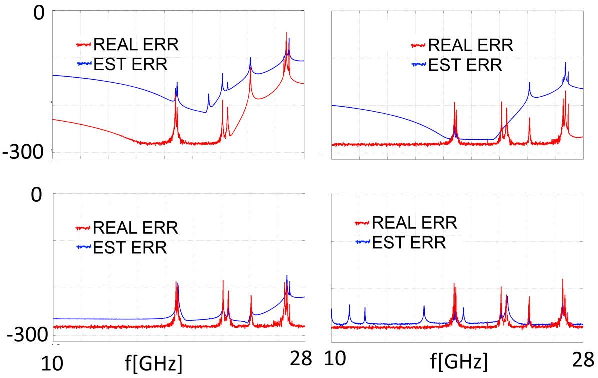
Error estimation (2)

- frequency band: (10,28) GHz



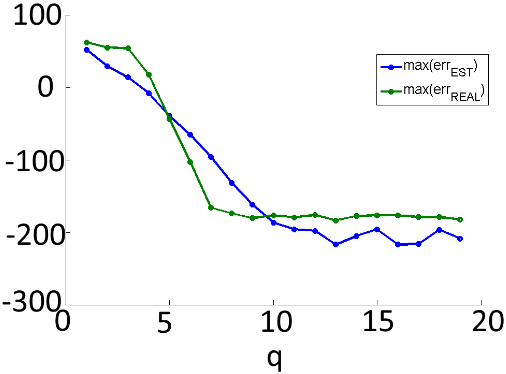
Error estimation (3)

- q = 5,7,9,12
- Real error vs estimation



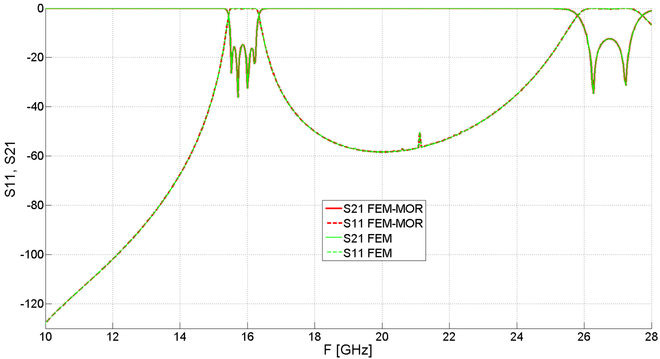
Error estimation (4)

- Integrated plots of real and estimated errors.
- Real error vs estimation



Error estimation (5)

- max(errEST(k)) = 10e-3
- Automatic selection of q = {8; 5; 6; 6; 9 }



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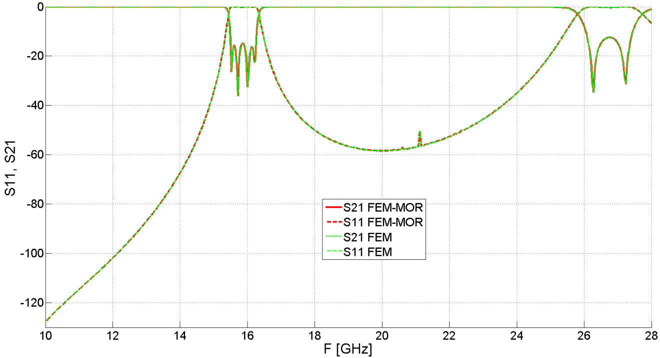

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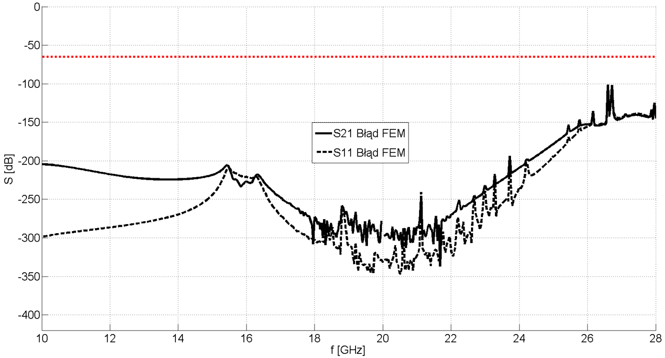
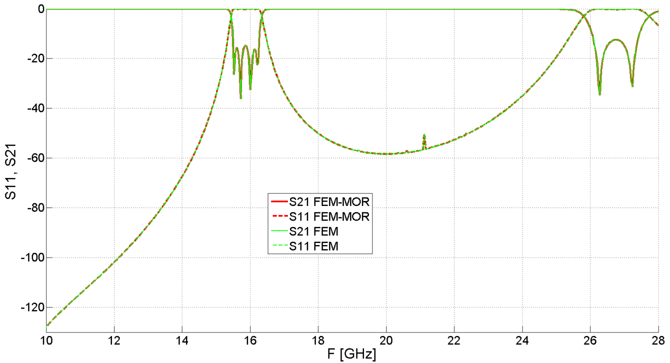


Error estimation (6)

- max(errEST(k)) = 10e-6
- Automatic selection of q = {12; 8; 9; 8; 12}



Error estimation (6)

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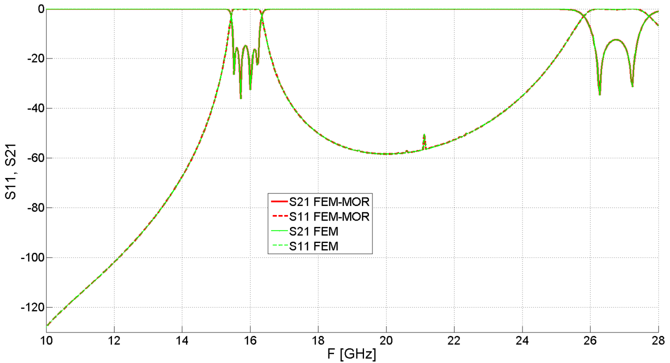
- Automatic selection of q = {12; 8; 9; 8; 12}



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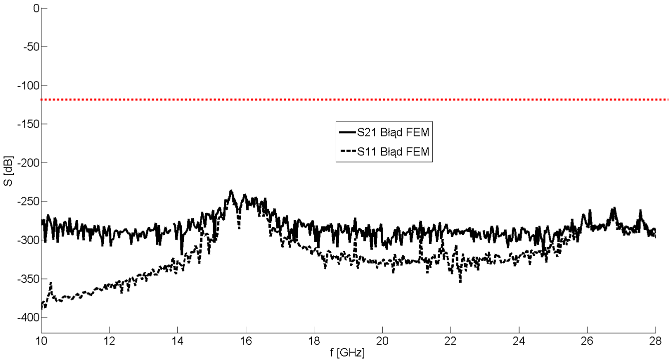

Conclusions

- Numerical experiments prove that incorporating reduced - order models into standard 3-D FEM algorithm results in significant reduction of number of variables and time cost, without sacrificing computational accuracy.
- These features make it a promising methodology for improving efficiency of finite element analysis.

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Conclusions

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Thank you for your attention