Modeling and Optimal Design of dipole and sextupole electromagnets for particle accelerators within the FAIR project

PhDs Eng. DAN Daniel

dan@lmn.pub.ro

http://www.lmn.pub.ro/ppl_dan.html

Numerical Modelling Laboratory

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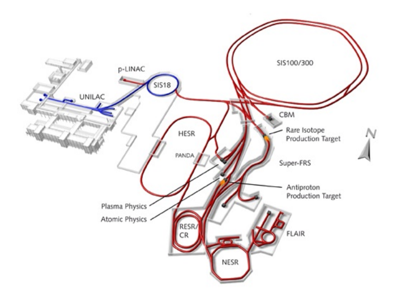
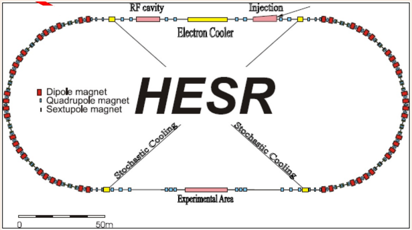
What is FAIR?

Facility for Antiproton and Ion Research



The FAIR complex



FAIR
Facility for Antiproton
and Ion Research
in Europe GmbH

Parties involved

in the FAIR project

Figure: The parties involved

...and in my professional training

Figure: The parties involved

Magnets

Sextupole and Steerer

- 66 Sextupoles;
- 53 Steerer magnets (horizontal and vertical)

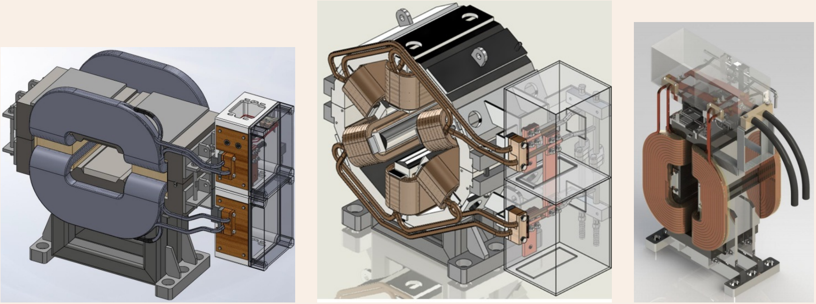


Figure: The magnets

Magnets

Parameters of the Sextupole and Steerer Vertical

Max deflection angle	2mrad at $p_{\max}$	
Aperture (diameter)	100mm	140mm
Magnetic length	300mm	300mm
Iron yoke length	270mm	270mm
Iron yoke width	580mm	450mm
Iron yoke height	450mm	450mm
Mass of iron (magnetic circuit)	$\approx$ 160kg	$\approx$ 160kg
Number of coils	2	6
Windings / coil	44	15
Layers / coil	4	2
Windings / layer	11mm	7.5mm
Conductor dimensions	$10.6 \times 7\text{mm}^2$	$10.6 \times 7\text{mm}^2$
Cooling bore	4mm	4mm
Cooper crossection	66.77mm^2	66.77mm^2
Length of conductor /coil	$\approx$ 72m	$\approx$ 12m
Cooper mass /coil	$\approx$ 39kg	$\approx$ 6.5kg
Current	304.1A	290A
Current density	5A/mm^2	4.77A/mm^2
Total mass	$\approx$ 350kg	$\approx$ 220kg
Voltage (DC)	12.84V	6.12V
Resistance	42.2mΩ	21.12mΩ
Inductivity	0.28mH	3.4mH
Power (DC)	3.9kW	1.8kW

Steerer magnet

Magnetic pole shape optimization

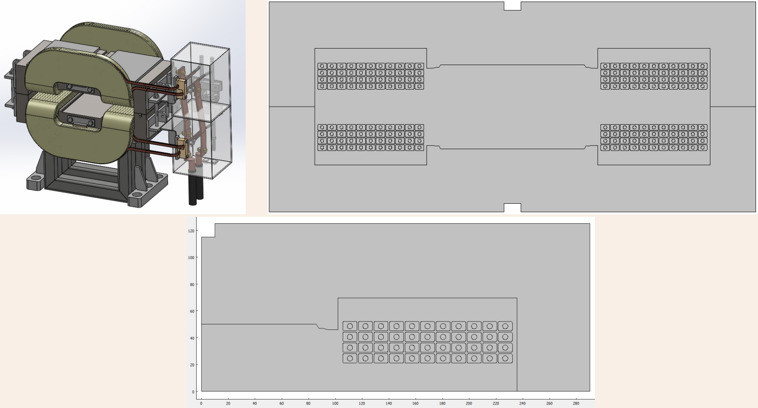


Figure: 3D SolidWorks model of the horizontal Steerer magnet (up - left). Cross section of the Steerer horizontal magnet (up - right). 1/4 of the cross section - Quadrant I

Steerer magnet

Magnetic pole shape optimization

The ideal pole shape and the good field region
For normal field, we have the equation for ideal (infinite) poles $y = \pm d/2$.

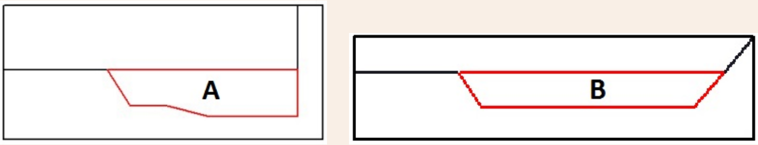


Figure: Shim Detail

Local homogeneity curve (LHC) of the field along the x-axis:

$$f_1(x) = \frac{\Delta B}{B_0}(x) = \frac{B(x,y) - B_0}{B_0}. \tag{1}$$

Horizontal measurement width means *width of good field region*.
The good field region is defined at radius: $R_{goodfield} = 33mm$.

Steerer magnet

Magnetic pole shape optimization

Geometric model. Modelling hypotheses.

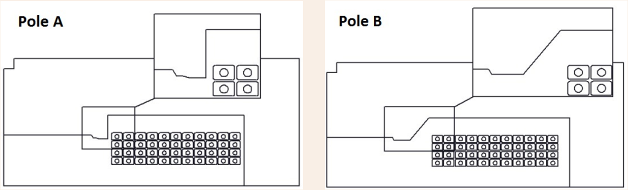


Figure: Cross section of the Steerer horizontal magnet with the poles shape optimization.

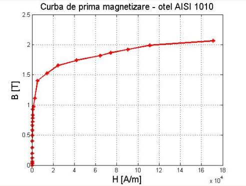


Figure: First magnetization curve

Steerer magnet

Magnetic pole shape optimization

Mathematical model

For this problem we have the equations of the magnetostatic regime with currents:

- Gauss's law for magnetism:
$$\nabla \mathbf{B} = 0, \tag{2}$$
where $\mathbf{B}(x,y) = \mathbf{i}B_x(x,y) + \mathbf{j}B_y(x,y)$ is the magnetic flux density, with $\mathbf{B} : \Omega \to \mathbb{R}^2$.
- Ampère's circuital law:
$$\nabla \times \mathbf{H} = \mathbf{J}; \tag{3}$$
where $\mathbf{H} : \Omega \to \mathbb{R}^2$ is magnetic field strength, and $\mathbf{J}(x,y) = \mathbf{k}J(x,y)$ is the current density, with $J : \Omega \to \mathbb{R}$
- The definition (not constitutive relations) of the auxiliary field is:
$$\mathbf{B} = \mu_0 \mu_r \mathbf{H}, \tag{4}$$
where $\mu_0 = 4\pi 10^{-7}\mathrm{NA}^{-1}$ is the permeability of free space and μ_r is the relative permeability.

Steerer magnet

Magnetic pole shape optimization

Mathematical model

Equation (4), allows the definition of magnetic vector potential $\mathbf{A}(\mathbf{r})$:

$$\mathbf{B} = \nabla \times \mathbf{A}, \quad \text{with} \quad \nabla \mathbf{A} = 0, \tag{5}$$

Consequently, magnetic vector potential is the solution of elliptic second order partial differential equations:

$$\nabla \times (\nu \nabla \times \mathbf{A}) = \mathbf{J}. \tag{6}$$

where $\nu = 1/\mu$ si the material constant. Also $\mathbf{J}\,[0, 0, J(x, y)]$ and $\mathbf{A}\,[0, 0, A(x, y)]$.
In 2D plan parallel problems we have the Poisson scalar equation:

$$\frac{\partial}{\partial x}\left(\nu \frac{\partial A}{\partial x}\right) + \frac{\partial}{\partial y}\left(\nu \frac{\partial A}{\partial y}\right) = -J, \tag{7}$$

where $A(x, y)$ is non-zero component of magnetic vector potential..
The uniqueness of the solution is given by internal source field (which is the current density), material properties and boundary conditions :

- Dirichlet boundary condition: $A \times n = 0 \Rightarrow \mathbf{B} \cdot \mathbf{n} = 0$ on $S_B \in \partial\Omega$;
- Neumann boundary condition: $\mathbf{H}_t = 0 \Leftrightarrow -dA/dn = B_{rt}$ on $S_H \in \Omega - S_B$.

Steerer magnet

Magnetic pole shape optimization

Numeric approach

Numerical solution was obtained by solving the problem: Find $A \in H_h$ such that

$$a(A, V) = f(V), \forall V \in H_h. \tag{8}$$

The expression (8) is weak form of a system of linear equations:

$$\mathbf{v}^T\mathbf{A}\mathbf{f} = \mathbf{v}^T\mathbf{f} \Rightarrow \mathbf{A}\mathbf{u} = \mathbf{f}, \quad \mathbf{u}, \mathbf{v}, \mathbf{f} \in \mathbb{R}^N, \ \mathbf{A} \in \mathbb{R}^{N\times N}, \tag{9}$$

where: $\mathbf{A} = \left[a\left(\varphi_j, \varphi_i\right)\right], \mathbf{f} = \left[f\left(\varphi_j\right)\right]$ and $\mathbf{u} = \left[u_i^h\right]$. The Matrix $\mathbf{A}$ - symmetric positive definite matrix and rare. Numerical solution:

$$A(x, y) = \mathbf{u}^T\varphi = \sum_{k=1}^{N} u_k\varphi_k(x, y). \tag{10}$$

Steerer magnet

Magnetic pole shape optimization

Numeric approach

Table: N is the number of triangles (mesh elements), D is number of degrees of freedom, the computation time is t. p is 3rd order Lagrange polynomial

Variant	T	D	t[s]	p	$f_{1,max}$%
A	56707	255460	29	3	0.04
B	56366	253927	28	3	0.03
No shim	54620	27393	31	3	0.20

Post processing. Results

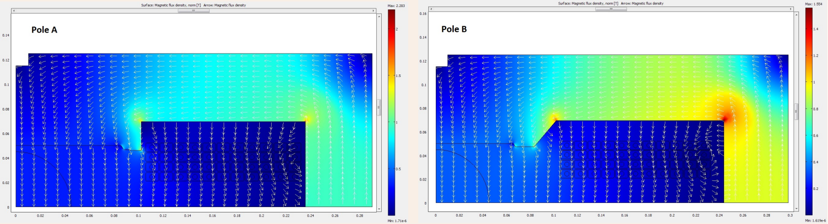


Figure: Magnetic flux density distribution map for versions A and B

Steerer magnet

Magnetic pole shape optimization

Post processing. Results

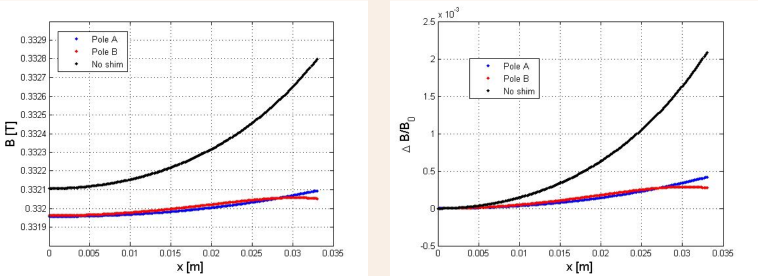


Figure: Variation of magnetic flux density vs. x. Local homogeneity curve vs. x

Sextupole magnet

Magnetic pole shape optimization

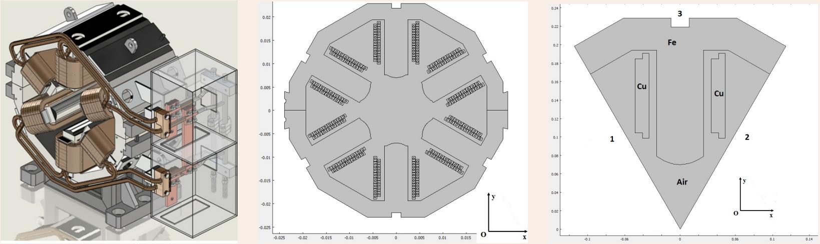


Figure: 3D model of the sextupole magnet (left). Cross section of the sextupole magnet after the pole shape optimization (center). Cross section of one sixth of the sextupole magnet (right)

Sextupole magnet

Magnetic pole shape optimization

Ideal pole shape and Field quality

The pole shape is described by a hyperbolic equation: $3x^2y - y^3 = \pm R_{\max}^3$, with $R_{\max} = 70$mm.

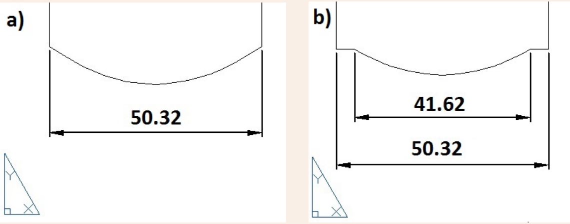


Figure: Shapes poles: a) Version V_1 - without shim ; b) Version V_2 - with shim.

The flux density deviations along the boundary of the defined good field region: $\Delta B/B = \left(B_i - \overline{B}\right)/\overline{B}$.

The good field region: $(2/3) \cdot d = (2/3) \cdot 140 \cong 93\text{mm} \quad \Rightarrow \quad \mathrm{R_{gfr}} \cong 46\text{mm}$, where $d = 140$mm is interpole gap, and $R = d/2$ is the radius.

Sextupole magnet

Magnetic pole shape optimization

Two situations:

- $I_{\text{max}} = 290\text{A}$, $R = 46\text{mm}$, $\theta = 0 \div 90^0$ and $I = 93\text{A}$, $R = 0 \div 48\text{mm}$, $\theta = 90^0$;

The geometric modeling. Symmetries

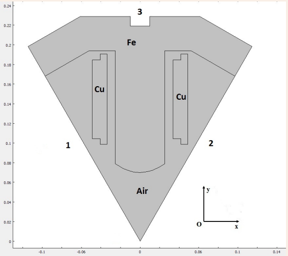


Figure: Cross section of one sixth of the sextupole magnet before the pole shape optimization

Physical modeling. Description of the problem The maximum current density is $J_{max} = 15 \cdot 290/0.001286 = 3.382582e6\mathrm{A/m^2}$, where $I_{max} = 290\mathrm{A}$, number of windings is 15, area of the cross section of coil $A_{Cu} = 0.001286\mathrm{m}^2$.

Sextupole magnet

Magnetic pole shape optimization

The mathematic model

2D parallel plane problem The stationary magnetic field equations:

$$div\mathbf{B} = 0; \quad rot\mathbf{H} = \mathbf{J}; \quad \mathbf{B} = \mathbf{f}\left(\mathbf{H}\right), \tag{11}$$

- Magnetic flux density: $\mathbf{B}(x,y) = \mathbf{i}B_x(x,y) + \mathbf{j}B_y(x,y)$, $\mathbf{B} : \Omega \rightarrow \mathbb{R}^2$;
- Magnetic field strength: $\mathbf{H}(x,y) = \mathbf{i}H_x(x,y) + \mathbf{j}H_y(x,y)$, $\mathbf{H} : \Omega \rightarrow \mathbb{R}^2$;
- Current density: $\mathbf{J}(x,y) = \mathbf{k}J(x,y)$, $J : \Omega \rightarrow \mathbb{R}$;
- Magnetization characteristic $\mathbf{f}(\mathbf{H}) = \mu_0\mu_r\mathbf{H}$ with $\mathbf{f} : \mathbb{R}^2 \rightarrow \mathbb{R}^2$.

Table: Data for first magnetization curve (**BH**).

$B[\mathrm{T}]$	0.12	1.45	1.70	1.80	2.0
$H[\mathrm{A/m}]$	50	700	4000	8000	25000

Each subdomain will be assumed to be homogeneous, having the same source field in all of its points.

Sextupole magnet

Magnetic pole shape optimization

The mathematic model

The potential magnetic vector and the Coulomb calibration:

$$\mathbf{B} = \nabla \times \mathbf{A}; \quad \nabla \mathbf{A} = 0. \tag{12}$$

The nonlinear second order differential equation:

$$\nabla \times \mathbf{g}\left(\nabla \times \mathbf{A}\right) = \mathbf{J}; \tag{13}$$

where $\mathbf{g} : \mathbb{R}^2 \to \mathbb{R}^2$ is the inverse of $\mathbf{f}$ function, presumed as bijective, so that $\mathbf{f} \circ \mathbf{g}$ represents the identity function.
The generalized Poisson's equation div-grad type:

$$\nabla \cdot \mathbf{g}\left(\nabla \cdot A\right) = J \Rightarrow \nabla \cdot \left(\nu \nabla \cdot A\right) = J_t. \tag{14}$$

with $\mathbf{A}(x, y) = \mathbf{k}A(x, y)$, $A : \Omega \to \mathbb{R}$.
Boundary conditions:

$$\mathbf{B}_n = \mathbf{n} \cdot \mathbf{B} \Leftrightarrow A = 0 \quad \text{on} \quad S_B \subset \partial\Omega; \tag{15}$$

$$\mathbf{H}_t = 0 \Leftrightarrow -\mathrm{d}A/\mathrm{d}n = B_{rt} \quad \text{on} \quad S_H = \Omega - S_B. \tag{16}$$

Sextupole magnet

Magnetic pole shape optimization

Numerical approach

Table: N is the number of triangles (mesh elements), D is number of degrees of freedom, the computation time is t, B is the magnetic flux density at value $R = 46\text{mm}$ and $\theta = 90^0$, $I = 290\text{A}$

Version	T	D	$t[\text{s}]$	$B[\text{T}]$
V_1	35547	161695	37	0.0982967
V_2	36689	166825	37	0.1024298

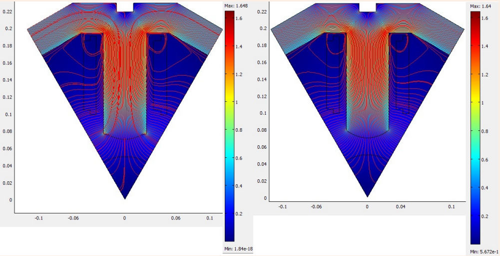


Figure: Magnetic flux density distribution map for Version V_1. Magnetic flux density distribution map for Version V_2

Sextupole magnet

Magnetic pole shape optimization

Results and conclusions

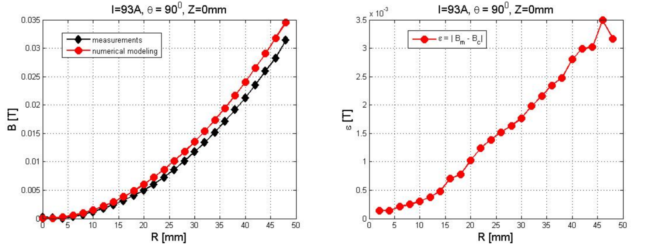


Figure: Variation of magnetic flux density vs. R. Absolute error of the measured and calculated data

The measured magnetic flux density corresponds from the point of view of the maximum values calculated for the value $I = 93\text{A}$ of the test current, in the center of the air gap. In second figure the absolute error of the measured and calculated data is plotted. The highest value is $3.5 \cdot 10^{-3}\text{T}$.

Sextupole magnet

Magnetic pole shape optimization

Results and conclusions

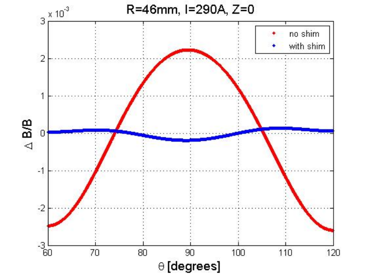


Figure: Relative deviation vs. θ

The maximum field deviation for the optimized pole shape is $1.3 \cdot 10^{-4} \leq 4 \cdot 10^{-4}$ within the "good field area" of $R = 46$mm for $B = 0.1024$T. Also, the maximum field deviation for the non-optimized pole shape is $2.2 \cdot 10^{-3} > 4 \cdot 10^{-4}$ within the "good field area" of $R = 46$mm for $B = 0.0983$T, where $4 \cdot 10^{-4}$ is the setpoint.